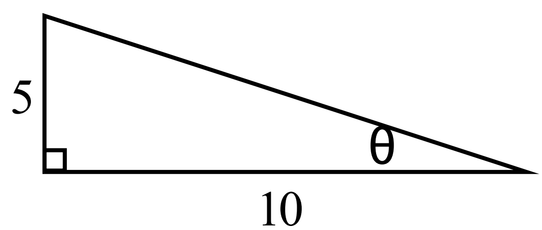
10-2 Solving Right Triangles

Trig Functions and their inverses

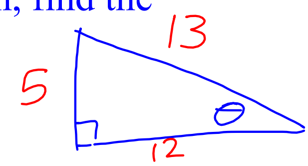
Functions	Inverses
Solve for sides	Solve for angles
$\sin\theta = x$	$\sin^{-1} x = \theta$
$\cos\theta = x$	$\cos^{-1} x = \theta$
$\tan\theta = x$	$\tan^{-1} x = \theta$

Reciprocal Functions	Reciprocal Inverses
$\csc\theta = x$	$\csc^{-1} x = \theta$
$\sec\theta = x$	$\sec^{-1} x = \theta$
$\cot\theta = x$	$\cot^{-1} x = \theta$



Find all six trig ratios for the given triangle:

Given the following trig function, find the remaining 5 functions:



$\csc\theta = \frac{13}{5}$ h o

$\sin\theta = 5/13$

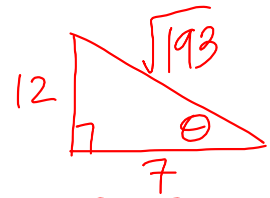
$\cos\theta = 12/13$

$\tan\theta = 5/12$

$\sec\theta = 13/12$

$\cot\theta = 12/5$

$\cot\theta = \frac{7}{12}$ a o



$\sin\theta = \frac{12}{\sqrt{193}}$

$\cos\theta = \frac{7}{\sqrt{193}}$

$\tan\theta = 12/7$

$\csc\theta = \frac{\sqrt{193}}{12}$

$\sec\theta = \frac{\sqrt{193}}{7}$

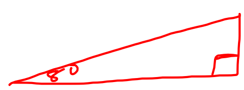
$12^2 + 7^2 = c^2$

$193 = c^2$

$c = \sqrt{193}$

Using your calculator, find:

$\tan 8° = 0.141$

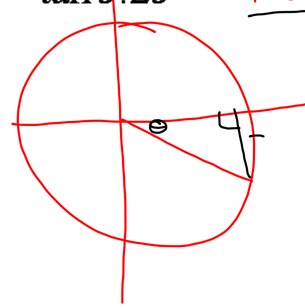


$\cot\frac{\pi}{12} = \frac{1}{\tan \pi/12}$

3.732

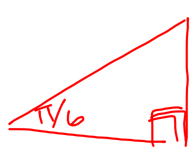
$\cos 18.15° = 0.950$

$\tan 5.25 = -1.677$



$\sec\frac{\pi}{6} = \frac{1}{\cos \pi/6}$

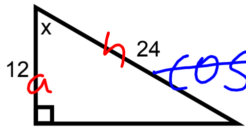
$\boxed{1.155}$



Inverse Trig

Find the measure of the indicated angle to the nearest **degree** (hint: calculator mode)

a)

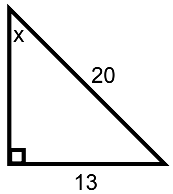


$$\cos^{-1}(\cos x) = \cos^{-1}\left(\frac{12}{24}\right)$$

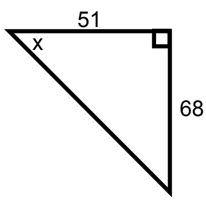
$$x = \cos^{-1}(12/24)$$

$$x = 60^\circ$$

b)



c)



To "solve" a triangle means to find ALL side lengths and angle measures.

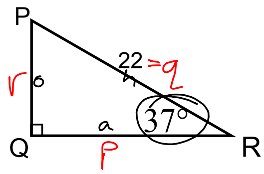
<u>REMEMBER</u>

-Triangles have an angle sum of 180 degrees

-Pythagorean Theorem to find a missing side when you know two

-Inverse Trig to find a missing angle

Solve each right triangle. Round lengths to the nearest ~~tenth~~ and angles to the nearest degree.



$\measuredangle P = 53^\circ$ $p = 17.6$

$\measuredangle Q = 90^\circ$ $q = 22$

$\measuredangle R = 37^\circ$ $r = 13.2$

$22 \cdot \sin(37^\circ) = \frac{r}{22} \cdot \cancel{22}$

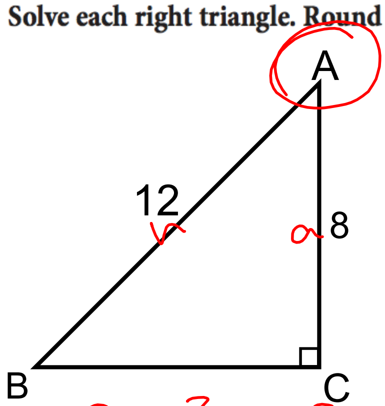
$r = 13.2$

$22 \cdot \cos 37^\circ = \frac{p}{22} \cdot 22$

$\sec\theta = \frac{12}{5}$ $\cos\theta = 5/12$

$= \cos^{-1}(5/12)$

Solve each right triangle. Round lengths to the nearest tenth and angles to the nearest degree.



$\measuredangle A = 48^\circ$ $a = 8.9$

$\measuredangle B = 42^\circ$ $b = 8$

$\measuredangle C = 90^\circ$ $c = 12$

$a^2 + 8^2 = 12^2$

$\sqrt{a^2} = \sqrt{80}$

$\boxed{a = 8.9}$

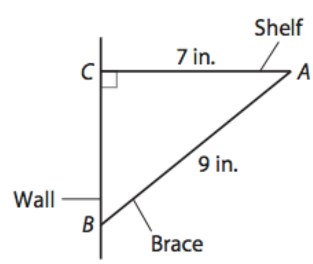
$\cos A = 8/12$

$A = \cos^{-1}(8/12)$

$180 - 90 - 48 = 42^\circ$

A building casts a 33-m shadow when the Sun is at an angle of 27° to the vertical. How tall is the building, to the nearest meter? How far is it from the top of the building to the tip of the shadow? What angle does a ray from the Sun along the edge of the shadow make with the ground?

A shelf extends perpendicularly 7 in. from a wall. You want to place a 9-in. brace under the shelf, as shown. To the nearest tenth of an inch, how far below the shelf will the brace be attached to the wall? To the nearest degree, what angle will the brace make with the shelf and with the wall?



Find the exact value. Find ***ALL*** possible solutions. (Hint: Use your unit circle!)

$\sin\left(\boxed{\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)}\right)$

$\sin\left(\frac{\pi}{6}\right) = \boxed{\frac{1}{2}}$

$\sin\left(\frac{7\pi}{6}\right) = \boxed{-\frac{1}{2}}$

$\cos\left(\sec^{-1} 2\right)$

$\cancel{\cos}\left(\cos^{-1} \frac{1}{2}\right)$

$\boxed{\frac{1}{2}}$

$\sin^{-1}\left(\cos\frac{\pi}{3}\right)$

$\sin^{-1}\left(\frac{1}{2}\right)$

$30^\circ, 150^\circ$

$\frac{\pi}{6}, \frac{5\pi}{6}$

$\cos^{-1}\left(\sin\frac{3\pi}{2}\right)$

$\cos^{-1}(-1)$

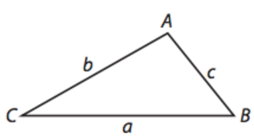
π

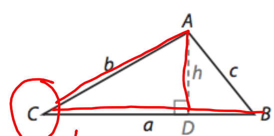
180°

Explore Deriving an Area Formula

You can use trigonometry to find the area of a triangle without knowing its height.

(A) Suppose you draw an altitude $\overline{AD}$ to side $\overline{BC}$ of $\triangle ABC$. Then write an equation using a trigonometric ratio in terms of $\angle C$, the height h of $\triangle ABC$, and the length of one of its sides.





$b \cdot \sin C = \frac{h}{b} \cdot b$

(B) Solve your equation from Step A for h.

$b\sin C = h$

(C) Complete this formula for the area of $\triangle ABC$ in terms of h and another of its side lengths: Area $= \frac{1}{2}$ $a h$

(D) Substitute your expression for h from Step B into your formula from Step C.

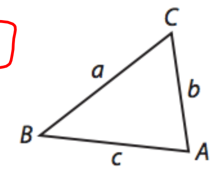
$\boxed{A = \frac{1}{2} a \cdot b\sin C}$

2. Suppose you used a trigonometric ratio in terms of $\angle B$, h, and a different side length. How would this change your findings? What does this tell you about the choice of sides and included angle?

Area formulas of a non-right triangles

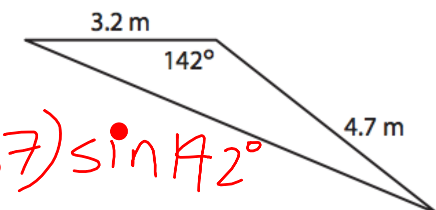
Area Formula for a Triangle in Terms of its Side Lengths

The area of $\triangle ABC$ with sides a, b, and c can be found using the lengths of two of its sides and the sine of the included angle: $\boxed{\text{Area} = \frac{1}{2}bc \sin A}$, $\boxed{\text{Area} = \frac{1}{2}ac \sin B}$, or $\boxed{\text{Area} = \frac{1}{2}ab \sin C}$.



Example 1 Find the area of each triangle to the nearest tenth.

(A)



$A = \frac{1}{2}(3.2)(4.7)\sin 142°$

$A = 4.6 \text{ m}^2$

B

$A = \frac{1}{2} \cdot 12 \cdot 15 \cdot \sin 34$

$A = 50.3 \text{ mm}^2$