March 21, 2017

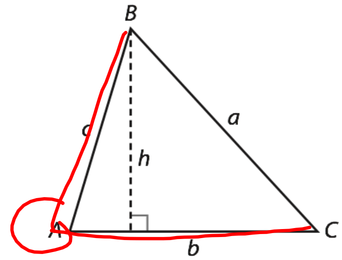
10-3 Law of Sines

Explore Use an Area Formula to Derive the Law of Sines

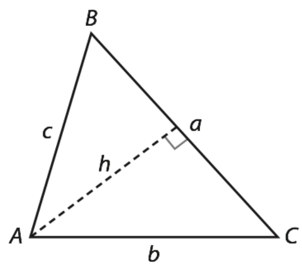
Recall that the area of a triangle can be found using the sine of one of the angles.

Area $= \frac{1}{2}b \cdot c \cdot \sin(A)$

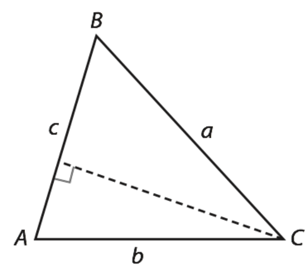
You can write variations of this formula using different angles and sides from the same triangle.



(A) Rewrite the area formula using side length a as the base of the triangle and $\angle C$.



(B) Rewrite the area formula using side length c as the base of the triangle and $\angle B$.



(C) What do all three formulas have in common?

(D) Why is this statement true?

$\frac{1}{2}b \cdot c \cdot \sin(A) = \frac{1}{2}a \cdot b \cdot \sin(C) = \frac{1}{2}c \cdot a \cdot \sin(B)$

(E) Multiply each area by the expression $\frac{2}{abc}$. Write an equivalent statement.

$$\frac{2}{\cancel{abc}}\left(\frac{1}{\cancel{2}}\cancel{b}\cancel{c}\cdot\sin A\right) = \frac{2}{\cancel{abc}}\left(\frac{1}{\cancel{2}}\cancel{a}\,b\sin C\right) = \left(\frac{1}{\cancel{2}}\cancel{c}\cancel{a}\sin B\right)\frac{2}{\cancel{abc}}$$

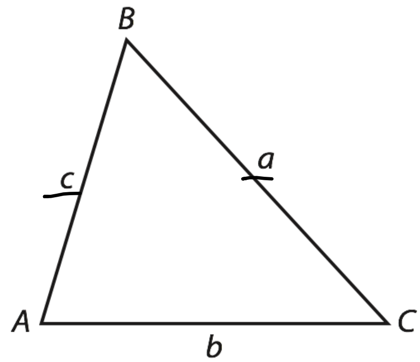
$$\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$$

Law of Sines

Law of Sines

Given: $\triangle ABC$

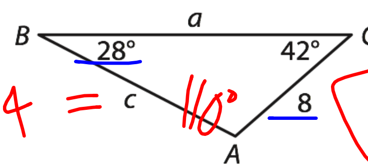
$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



Your Turn

Find all the unknown measures using the given triangle. Round to the nearest tenth.

4.



$180-42-28$

$\boxed{A=110°}$

$11.4 = c$

$$\frac{\sin 28°}{8} = \frac{\sin 42°}{c}$$

$$\frac{c\cancel{\sin 28°}}{\cancel{\sin 28°}} = \frac{8\sin(42°)}{\sin 28°}$$

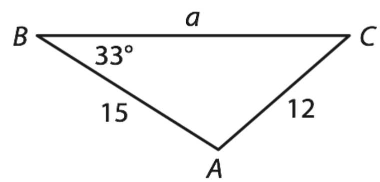
$$\boxed{c = 11.4}$$

$$\frac{\sin 28°}{8} = \frac{\sin 110°}{a}$$

$$\frac{a\cancel{\sin 28}}{\cancel{\sin 28}} = \frac{8\sin 110°}{\sin 28}$$

$$\boxed{a = 16}$$

5.



$$\frac{\sin 33°}{12} = \frac{\sin C}{15}$$

$$\frac{15\sin 33°}{12} = \frac{12\sin C}{12}$$

$$\sin^{-1}\left(\frac{15\sin 33°}{12}\right) = \cancel{\sin^{-1}}(\sin C)$$

$$\boxed{42.9° = C}$$

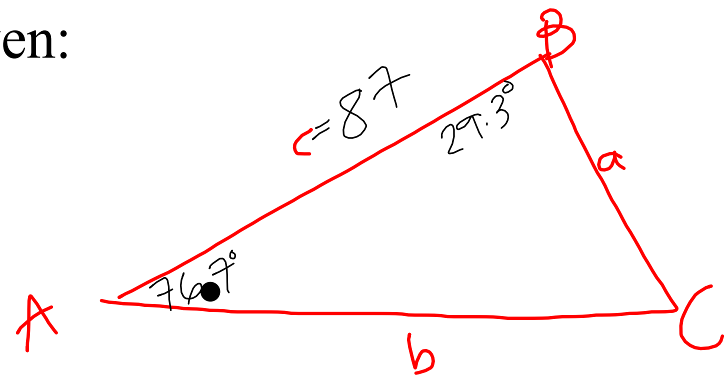
$$\boxed{A = 104.1°}$$

$$\frac{\sin 33}{12} = \frac{\sin 104.1°}{a}$$

$$\frac{a\cancel{\sin 33}}{\cancel{\sin 33}} = \frac{12\sin 104.1}{\sin 33}$$

$$\boxed{a = 21.4}$$

Solve the triangle given:
A = 76.7°
B = 29.3°
c = 87



C = 74°
b = 44.3
a = 88.1

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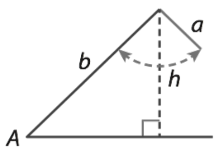
Explain 2 Evaluating Triangles When SSA is Known Information

When you use the Law of Sines to solve a triangle for which you know side-side-angle (SSA) information, zero, one, or two triangles may be possible. For this reason, SSA is called the ambiguous case.

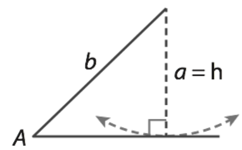
Ambiguous Case

Given a, b, and $m\angle A$.

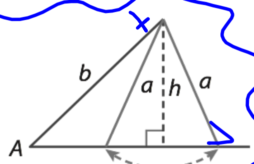
$\angle A$ is acute.



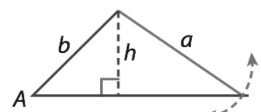
$a < h$
No triangle



$a = h$
One triangle

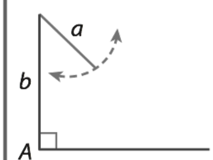


$h < a < b$
Two triangles

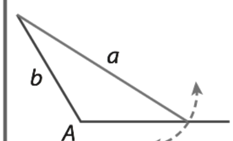


$a \geq b$
One triangle

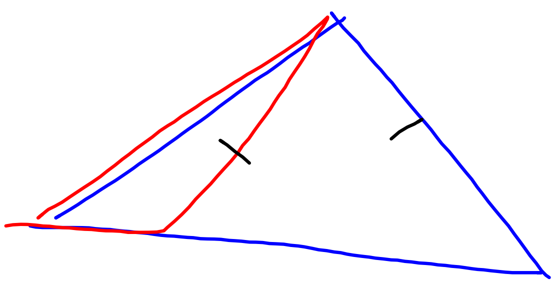
$\angle A$ is right or obtuse.



$a \leq b$
No triangle



$a > b$
One triangle



Solve the triangle ABC.

Given a=20, b=5, B=42°

$$\frac{\sin 42}{5} = \frac{\sin A}{20}$$

$\boxed{\text{NO } \Delta}$

Solve the triangle ABC.

Given: $a=3$, $b=2$, $A=40^{o}$

Solve the triangle ABC.

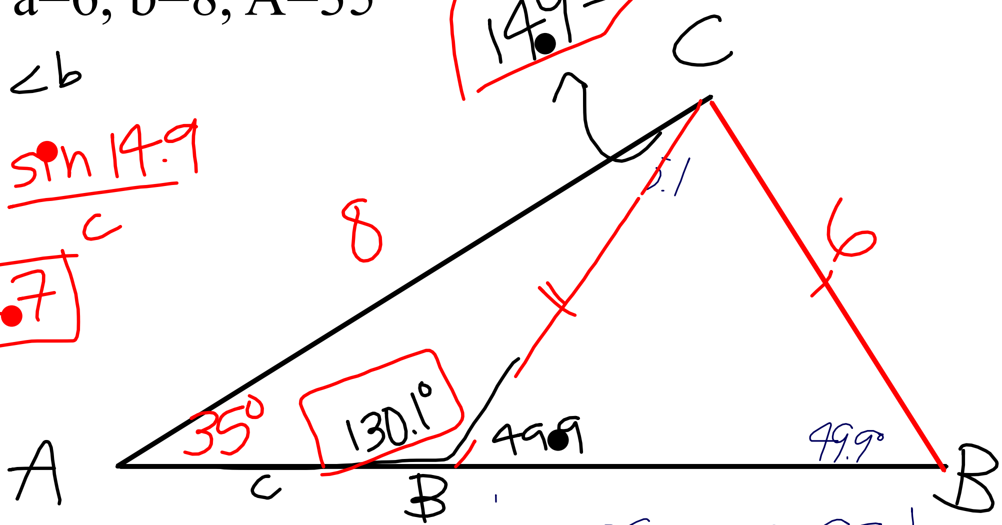
Given: a=6, b=8, A=35°

$a < b$

$\boxed{14.9° = C}$

$$\frac{\sin 35}{6} = \frac{\sin 14.9}{c}$$

$$\boxed{c = 2.7}$$



$$\frac{\sin 35}{6} = \frac{\sin B}{8}$$

$$\frac{8 \sin 35°}{6} = \frac{\cancel{6} \sin B}{\cancel{6}}$$

$$\sin^{-1}\left(\frac{8 \sin 35°}{6}\right) = \cancel{\sin^{-1}}\left(\cancel{\sin} B\right)$$

$$\boxed{49.9° = B}$$

$$\frac{\sin 35}{6} = \frac{\sin 95.1}{c}$$

$$\frac{c \cancel{\sin 35}}{\cancel{\sin 35}} = \frac{6 \sin 95.1}{\sin 35}$$

$$\boxed{c = 10.4}$$

$$\boxed{C = 95.1°}$$

Solve the triangle ABC.

Given: $a=37$, $b=40$, $A=71^{\circ}$