11-1 Pythagorean Identities

Identity:

equality that is true for all values of the domain for both expressions as long as they are both defined

$$\tan\theta \cdot \cos\theta = \sin\theta$$

this is true for all θ, as long as $\sin\theta$, $\cos\theta$, and $\tan\theta$ are defined

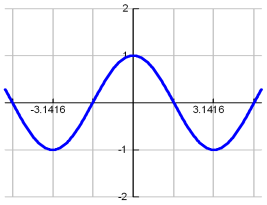
Reciprocal & Quotient Relationships

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

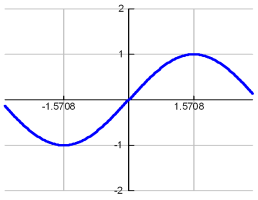
$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Odd/Even Identities



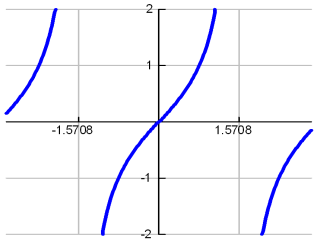
$$\cos(-x) = \cos x$$

$$\sec(-x) = \sec x$$



$$\sin(-x) = -\sin x$$

$$\csc(-x) = -\csc x$$

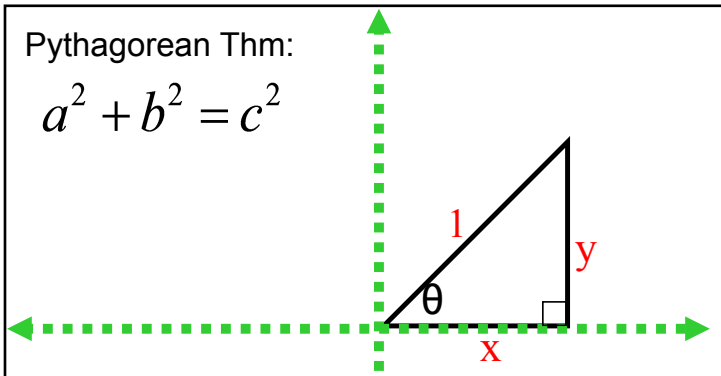


$$\tan(-x) = -\tan x$$

$$\cot(-x) = -\cot x$$

Pythagorean Thm:

$a^2 + b^2 = c^2$



$x^2 + y^2 = 1^2$

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\cos^2\theta + \sin^2\theta = 1$

Can we write an equation about the relationship of x, y, and r?

$\sin\theta = \frac{y}{1} = y$

$\cos\theta = \frac{x}{1} = x$

Pythagorean Relationships

$$\sin^2\theta + \cos^2\theta = 1$$

- $\sin^2\theta = 1 - \cos^2\theta$
- $\cos^2\theta = 1 - \sin^2\theta$

Now lets divide by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Pythagorean Relationships

$$1 + \tan^2\theta = \sec^2\theta$$

- $1 = \sec^2\theta - \tan^2\theta$
- $\tan^2\theta = \sec^2\theta - 1$

Now lets divide by $\sin^2\theta$

$$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$* \quad 1 + \cot^2\theta = \csc^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 = \csc^2\theta - \cot^2\theta$$

$$\cot^2\theta = \csc^2\theta - 1$$

Pythagorean Relationships

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Simplify:

$\cot x \tan x$

$\frac{1}{\tan x} \cdot \frac{\tan x}{1}$

$\frac{\tan x}{\tan x} = 1$

* $\sin\theta \csc\theta$

$\sin\theta \cdot \frac{1}{\sin\theta} = 1$

$\sin x \csc(-x)$

$\sin x(-\csc x)$

$\sin x\left(-\frac{1}{\sin x}\right)$

-1

$\frac{\sec^2 x}{\tan^2 x} = \sec^2 x \cdot \frac{1}{\tan^2 x}$

$\sec^2 x \cdot \cot^2 x$

$\frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x}$

$\frac{1}{\sin^2 x} = \csc^2 x$

Simplify

$$\frac{\cos x \sec x}{\cos x \sin x} - \frac{\sin x \cdot \sin x}{\cos x \sin x}$$

$$\frac{\cos x \sec x - \sin^2 x}{\cos x \sin x}$$

$$\frac{1 - \sin^2 x}{\cos x \sin x} = \frac{\cos^2 x}{\cos x \sin x} = \frac{\cos x}{\sin x} = \cot x$$

Simplify:

$$\frac{\sin\alpha + 1}{(\sin\alpha + 1)} \cdot \frac{1}{(\sin\alpha - 1)} + \frac{-1}{\sin\alpha + 1} \cdot \frac{(\sin\alpha - 1)}{\sin\alpha - 1}$$

$$\frac{\sin\alpha + 1 - \sin\alpha + 1}{\sin^2\alpha - \sin\alpha + \sin\alpha - 1} = \frac{2}{\sin^2\alpha - 1}$$

$$\frac{2}{\sin^2\alpha - (\sin^2\alpha + \cos^2\alpha)} = \frac{2}{-(-\sin^2\alpha + 1)}$$

$$\frac{2}{-\cos^2\alpha} = -2\sec^2\alpha$$

Establish the Identity:

$\csc x \cos x = \cot x$

$\frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$ ✓

$(1 - \sin^2 x)(1 + \tan^2 x) = 1$

$(\cos^2 x)(\sec^2 x)$

$(\cos^2 x)\left(\frac{1}{\cos^2 x}\right) = 1$ ✓

$\cos x(\tan x + \cot x) = \csc x$

$\cos x \tan x + \cos x \cot x$

$\cos x \frac{\sin x}{\cos x} + \cos x \cdot \frac{\cos x}{\sin x}$

$\frac{\sin x}{\sin x} \frac{\sin x}{1} + \frac{\cos^2 x}{\sin x}$

$\frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$ ✓