

Use basic identities to simplify the expression:

$$1. \frac{1 - \cos^2 \theta}{\sin \theta} = \frac{\sin^2 \theta}{\sin \theta} = \boxed{\sin \theta}$$

$$2. \frac{1 + \tan^2 x}{\csc^2 x} = \frac{\sec^2 x}{\csc^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\csc^2 x}$$

$$= \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} = \frac{\sin^2 x}{\cos^2 x} = \boxed{\tan^2 x}$$

$$3. \frac{\sin^2 u + \tan^2 u + \cos^2 u}{\sec u}$$

$$\frac{1 + \tan^2 u}{\sec u} = \frac{\sec^2 u}{\sec u}$$

$$\boxed{\sec u}$$

$$4. (\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x)$$

$$1 + \tan^2 x + 1 + \cot^2 x - \tan^2 x - \cot^2 x$$

$$\boxed{2}$$

$$5. \frac{1(1+\cos x)}{1-\cos x} \cdot \frac{1(1-\cos x)}{1+\cos x}$$

$$\frac{1+\cos x-1+\cos x}{1-\cos^2 x} = \frac{2\cos x}{\sin^2 x} = \frac{2\cot x}{\sin x} = \boxed{2\cot x \csc x}$$

$$6. (\sin x)(\tan x + \cot x)$$

$$\sin x \tan x + \sin x \cot x$$

$$\sin x \cdot \frac{\sin x}{\cos x} + \sin x \cdot \frac{\cos x}{\sin x}$$

$$\frac{\sin^2 x}{\cos x} + \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x} = \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$= \frac{1}{\cos x} = \boxed{\sec x}$$

$$7. \sin x \cdot \cos x \cdot \tan x \cdot \sec x \cdot \csc x$$

$$\frac{\sin x}{1} \cdot \frac{\cos x}{1} \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$\boxed{\tan x}$$

$$8. \frac{1}{\sin^2 x} + \frac{\sec^2 x}{\tan^2 x}$$

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{2}{\sin^2 x} = \boxed{2\csc^2 x}$$

$$9. \sec(-x)\cos(-x)$$

$$\frac{1}{\cos x} \cdot \frac{\cos x}{1} = \boxed{1}$$

$$10. \cot(-x)\tan(-x)$$

$$-\cot x (-\tan x)$$

$$-\frac{1}{\tan x} \cdot -\frac{\tan x}{1} = \boxed{1}$$

Establish each identity:

11.  $\csc\theta \cdot \tan\theta = \sec\theta$

$$\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta \checkmark$$

12.  $\sin\theta(\cot\theta + \tan\theta) = \sec\theta$

$$\begin{aligned} & \sin\theta \left( \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \right) \\ & \frac{\cos\theta}{\cos\theta} \frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta} \\ & \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta \checkmark \end{aligned}$$

13.  $\sin^2 x(1 + \cot^2 x) = 1$

$$\begin{aligned} & \sin^2 x (\csc^2 x) \\ & \frac{\sin^2 x}{1} \left( \frac{1}{\sin^2 x} \right) = 1 \checkmark \end{aligned}$$

14.  $\tan^2 \alpha \cos^2 \alpha + \cot^2 \alpha \sin^2 \alpha = 1$

$$\begin{aligned} & \frac{\sin^2 \alpha \cos^2 \alpha}{\cos^2 \alpha} + \frac{\cos^2 \alpha \sin^2 \alpha}{\sin^2 \alpha} \\ & \sin^2 \alpha + \cos^2 \alpha = 1 \checkmark \end{aligned}$$

15.  $\cot\beta + \tan\beta = \sec\beta \csc\beta$

$$\begin{aligned} & \frac{\cos\beta \cos\beta}{\sin\beta \cos\beta} + \frac{\sin\beta (\sin\beta)}{\cos\beta \sin\beta} \\ & \frac{\cos^2 \beta + \sin^2 \beta}{\sin\beta \cos\beta} = \frac{1}{\sin\beta \cos\beta} \\ & = \sec\beta \csc\beta \checkmark \end{aligned}$$

16.  $\sin^2 \alpha + \tan^2 \alpha + \cos^2 \alpha = \sec^2 \alpha$

$$1 + \tan^2 \alpha = \sec^2 \alpha \checkmark$$

17.  $\frac{\sec w}{\csc w} + \frac{\sin w}{\cos w} = 2 \tan w$

$$\begin{aligned} & \frac{\sin w}{\cos w} + \frac{\sin w}{\cos w} \\ & \tan w + \tan w = 2 \tan w \checkmark \end{aligned}$$

18.  $\frac{1 + \frac{\sin^2 x}{1 + \cos^2 x}}{1 + \cos^2 x} = \frac{2}{1 + \cos^2 x}$

$$\frac{1 + \cos^2 x + \sin^2 x}{1 + \cos^2 x} = \frac{2}{1 + \cos^2 x} \checkmark$$