11-2 Sum & Difference Identities

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Express the angle as a sum or difference of 2 special angles: $(0, 30, 45, 60, 90)$

135°

$45+90$

150°

$90+60$

Find the exact value of:

$\cos 105^\circ = \cos(45+60)$

$\cos 45\cos 60 - \sin 45 \sin 60$

$\frac{\sqrt{2}}{2}\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$

$\sin 15^\circ = \sin(45-30)$

$\sin 45\cos 30 - \cos 45 \sin 30$

$\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\tan 75^\circ$

$\tan(45+30)$

$$\frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} = \frac{1+\frac{\sqrt{3}}{3}}{1-1\cdot\frac{\sqrt{3}}{3}}$$

$$\frac{\frac{3}{3}\frac{1}{1} + \frac{\sqrt{3}}{3}}{\frac{3}{3}\frac{1}{1} - \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3} \div \frac{3-\sqrt{3}}{3}$$

$$= \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}}$$

$$= \boxed{\frac{3+\sqrt{3}}{3-\sqrt{3}}}$$

Write as the sin, cos, or tan of an angle:

$$\sin 50^\circ \cos 26^\circ - \cos 50^\circ \sin 26^\circ$$

$\sin(50-26) = \sin 24^\circ$

$$\cos 50^\circ \cos 26^\circ - \sin 50^\circ \sin 26^\circ$$

$\cos(50+26) = \cos 76^\circ$

$$\frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}$$

$\tan(60-45)$

$\tan 15^\circ$

Prove the identity:

$$\cos(x - 90°) = \sin x$$

$\cos x \cos 90 + \sin x \sin 90$

$\cancel{\cos x \cdot 0} + \sin x \cdot 1$

$\sin x$ ✓

$$\sin(x - y) + \sin(x + y) = 2 \sin x \cos y$$

$\sin x \cos y \cancel{- \cos x \sin y} + \sin x \cos y \cancel{+ \cos x \sin y}$

$2 \sin x \cos y$ ✓