

Write out the sum or difference identities

$$\sin(u+v) = \frac{\sin u \cos v + \cos u \sin v}{}$$

$$\sin(u-v) = \frac{\sin u \cos v - \cos u \sin v}{}$$

$$\cos(u+v) = \frac{\cos u \cos v - \sin u \sin v}{}$$

$$\cos(u-v) = \frac{\cos u \cos v + \sin u \sin v}{}$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Express the angle as a sum or difference of 2 special angles.

- | | | | |
|------------------------------|---------------|----------------|----------------|
| 1. 15° | 2. 75° | 3. 120° | 4. 105° |
| $60 - 45$
or
$45 - 30$ | $30 + 45$ | $60 + 60$ | $60 + 45$ |

Use the sum or difference identity to find an exact value.

5. $\tan 15^\circ = \tan(45 - 30)$

$$\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \frac{\sqrt{3}}{3}} = \frac{\frac{3 - \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

7. $\cos 75^\circ = \cos(30 + 45)$

$$\cos 30 \cos 45 - \sin 30 \sin 45 = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4} \text{ or } \frac{\sqrt{6} - \sqrt{2}}{4}$$

9. $\sin 105^\circ = \sin(60 + 45)$

$$\sin 60 \cos 45 + \cos 60 \sin 45 = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4} \text{ or } \frac{\sqrt{6} + \sqrt{2}}{4}$$

11. $\cos 15^\circ = \cos(45 - 30)$

$$\cos 45 \cos 30 + \sin 45 \sin 30 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

6. $\sin 75^\circ = \sin(30 + 45) = \sin 30 \cos 45 + \cos 30 \sin 45$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \text{ or } \frac{\sqrt{2} + \sqrt{6}}{4}$$

8. $\sin 135^\circ = \sin(90 + 45)$

$$\sin 90 \cos 45 + \cos 90 \sin 45 = 1 \cdot \frac{\sqrt{2}}{2} + 0 \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$$

10. $\tan 105^\circ = \tan(60 + 45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$

12. $\tan 120^\circ = \tan(60 + 60)$

$$\frac{\tan 60 + \tan 60}{1 - \tan 60 \tan 60} = \frac{\sqrt{3} + \sqrt{3}}{1 - \sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}}{1 - 3} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

Write as a function of one angle.

13. $\sin 40^\circ \cos 30^\circ + \cos 40^\circ \sin 30^\circ$

$$\sin 70^\circ$$

15. $\sin 50^\circ \cos 10^\circ - \cos 50^\circ \sin 10^\circ$

$$\sin 40^\circ$$

17. $\cos 38^\circ \cos 37^\circ + \sin 38^\circ \sin 37^\circ$

$$\cos 1^\circ$$

Prove the identity:

19. $\sin(x-90) = -\cos x$

$$\sin x \cos 90 - \cos x \sin 90$$

$$\sin x \cdot 0 - \cos x \cdot 1$$

$$\boxed{-\cos x} \checkmark$$

21. $\tan(\theta+45^\circ) = \frac{1+\tan\theta}{1-\tan\theta}$

$$\frac{\tan\theta + \tan 45^\circ}{1 - \tan\theta \tan 45^\circ}$$

$$\frac{\tan\theta + 1}{1 - \tan\theta} \checkmark$$

14. $\cos 15^\circ \cos 25^\circ - \sin 15^\circ \sin 25^\circ$

$$\cos 40^\circ$$

16. $\frac{\tan 19^\circ + \tan 47^\circ}{1 - \tan 19^\circ \cdot \tan 47^\circ}$

$$\tan 66^\circ$$

18. $\frac{\tan 7^\circ + \tan 18^\circ}{1 - \tan 7^\circ \cdot \tan 18^\circ}$

$$\tan 25^\circ$$

20. $\cos(x-90) = \sin x$

$$\cos x \cos 90 + \sin x \sin 90$$

$$\cos \cdot 0 + \sin x \cdot 1$$

$$\boxed{\sin x} \checkmark$$

22. $\cos(x-y) + \cos(x+y) = 2\cos x \cos y$

$$\cos x \cos y + \cancel{\sin x \sin y} + \cos x \cos y - \cancel{\sin x \sin y}$$

$$2\cos x \cos y \checkmark$$