Secondary Math III H
HW 11-3 Double angles

Name: Key
Period:

Write the following equations:

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

Given that $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ and $\tan\theta = \frac{3}{4}$ find the following

1. $\sin 2\theta$

$2\sin\theta\cos\theta$

$(\frac{3}{5})(\frac{4}{5})$

$\boxed{24/25}$

2. $\cos 2\theta$

$\cos^2\theta - \sin^2\theta$

$(4/5)^2 - (3/5)^2$

$\boxed{7/25}$

3. $\tan 2\theta$

$\dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2(3/4)}{1-9/16} = \dfrac{6/4}{7/16} = \frac{6}{4}\cdot\frac{16}{7} = \boxed{\frac{24}{7}}$

Given that $\sin\theta = \frac{\sqrt{5}}{5}$, $\cos\theta = \frac{2\sqrt{5}}{5}$ and $\tan\theta = \frac{1}{2}$ find the following

4. $\sin 2\theta$

$2\left(\frac{\sqrt{5}}{5}\right)\left(\frac{2\sqrt{5}}{5}\right)$

$\frac{20}{25} = \boxed{\frac{4}{5}}$

5. $\cos 2\theta$

$\frac{20}{25} - \frac{5}{25}$

$\frac{15}{25} = \frac{3}{5}$

6. $\tan 2\theta$

$\dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{1}{1-1/4} = 1 \div \frac{3}{4} = 1\cdot\frac{4}{3}$

$\boxed{4/3}$

Prove the following identities:

7. $\csc 2\theta = \dfrac{\sec\theta\csc\theta}{2}$

$\dfrac{1}{\sin 2\theta} = \dfrac{1}{2\sin\theta\cos\theta} = \boxed{\dfrac{\csc\theta\sec\theta}{2}}$ ✓

8. $\sec 2\theta = \dfrac{\sec^2\theta}{2-\sec^2\theta}$ (hint: use $\cos 2\theta = 2\cos^2\theta - 1$)

$\dfrac{1}{\cos 2\theta} = \dfrac{1}{(2\cos^2\theta - 1)}\cdot\dfrac{\sec^2\theta}{\sec^2\theta} = \boxed{\dfrac{\sec^2\theta}{2-\sec^2\theta}}$ ✓

Use the double angle formulas to find the exact value of the following:

9. $\cos 120° = \cos 2(60)$

$\cos^2(60) - \sin^2(60)$

$\left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{1}{4} - \frac{3}{4} = -\frac{2}{4} = \boxed{-\frac{1}{2}}$

10. $\sin 120° = \sin 2(60)$

$= 2\sin 60 \cos 60$

$= \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{3}}{2}}$

11. $\cos^2 15° - \sin^2 15°$

$\cos 30 = \boxed{\frac{\sqrt{3}}{2}}$

12. $2\sin 22.5° \cos 22.5°$

$\sin 45° = \boxed{\frac{\sqrt{2}}{2}}$

13. $\frac{2\tan 45°}{1-\tan^2 45°} = \tan 90°$

$\boxed{\text{undefined}}$

14. $2\cos^2 30° - 1$

$\cos 60° = \boxed{\frac{1}{2}}$

Assume that the $\cos 2x = \frac{1}{2}$ find the following using the power reducing formula:

15. $\cos^2 x$

$\frac{1+\cos 2x}{2} = \frac{\frac{2}{2}+\frac{1}{2}}{2}$

$\frac{3}{2} \cdot \frac{1}{2} = \boxed{\frac{3}{4}}$

16. $\sin^2 x$

$\frac{1-\cos 2x}{2}$

$\frac{1-\frac{1}{2}}{2} = \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{1}{4}}$

17. $\tan^2 x$

$\frac{1-\cos 2x}{1+\cos 2x} = \frac{1-\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{2} \div \frac{3}{2}$

$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} = \boxed{\frac{1}{3}}$

Use the power reducing formula to express the following in trig functions with no exponents.

18. $\sin^3 x$

$\sin x(\sin^2 x)$

$\sin x\left(\frac{1-\cos 2x}{2}\right)$

19. $\tan^3 x$

$\tan x(\tan^2 x)$

$\tan x\left(\frac{1-\cos 2x}{1+\cos 2x}\right)$