

2-2 Finite Geometric Series

You have 2 biological parents, 4 biological grandparents, and 8 biological great-grandparents. How many great-great-great-great grandparents (6th generation) do you have?

2
4

8 9

16 99

32 999

64 9999

64

How many direct ancestors do you have if you trace your ancestry back 6 generations?

126

Series: def: sum of the terms in a sequence

Sum: usually a total of a finite number of items added together

Summation

$$a_1 + a_2 + a_3 + \dots + a_n$$

(how do we write the sum of long lists of numbers?)

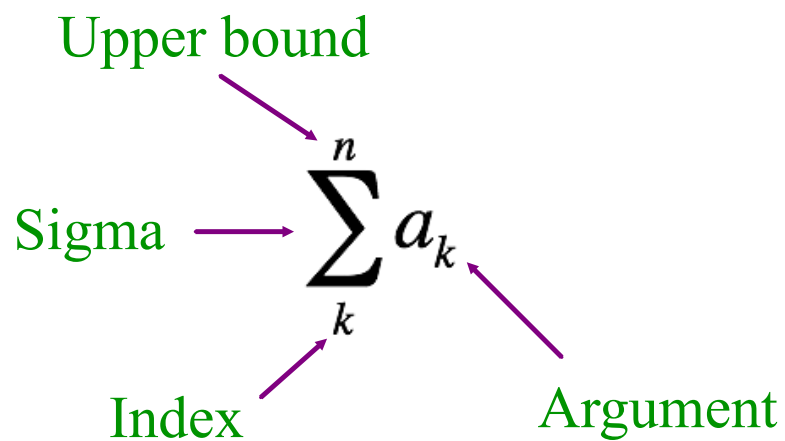
Σ sigma means summation

Summation notation: $\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$

Handwritten annotations:

- Upper bound (points to n)
- sigma (points to Σ)
- index (points to $k=1$)
- argument/ explicit eqn (points to a_k)

Vocabulary



Read as:

The sum of a_k from k to n

Formula for Finite Geometric Series

$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n = \frac{a_1(1-r^n)}{1-r}$$

Find the sum of the series:

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & +15 & +45 & +135 & +405 & +1215 \end{array}$$

$$\sum_{n=1}^6 5(3)^{n-1}$$

~~Your turn. 1-2+4-8+16-32~~

$$8+16+32+\dots+512$$

$$\frac{8(2)^{n-1}}{8} = \frac{512}{8}$$

$$(2)^{n-1} = \underline{64}$$

$$(2)^{n-1} = (2)^{\boxed{6}}$$

$$n-1=6$$

$$n = \boxed{7}$$

$$\sum_{n=1}^7 8(2)^{n-1} = \boxed{1016}$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{8} + \dots - \frac{1}{256}$$

$$\frac{2}{1} \cdot \frac{1}{2} \left(-\frac{1}{2}\right)^{n-1} = \frac{-1}{256} \cdot 2$$

$$\left(-\frac{1}{2}\right)^{n-1} = -\frac{1}{128}$$

$$\left(-\frac{1}{2}\right)^{n-1} = \left(-\frac{1}{2}\right)^{\boxed{7}}$$

$$n-1=7$$

$$n=8$$

$$\sum_{n=1}^8 \left(\frac{1}{2} \left(-\frac{1}{2}\right)^{n-1}\right) = \frac{85}{256}$$

Niobe is saving for a down payment on a new car, which she intends to buy a year from now. At the end of each month, she deposits \$200 from her paycheck into a dedicated savings account, which earns 3% annual interest that is applied to the account balance each month. After making 12 deposits, how much money will Niobe have in her savings account?

Sum of a Finite Arithmetic Sequence:

$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$$

$$= \frac{n(a_1 + a_n)}{2}$$

$$= \frac{n}{2}(2a_1 + (n-1)d)$$

Find the sum of the arithmetic sequence

~~8, 1, 6, 13, 20, 27, ...~~

1 2 3 13
117, 110, 103, ..., 33

$$117 - 7(n-1) = 33$$

$$117 - 7n + 7 = 33$$

$$124 - 7n = 33$$

$$n = 13$$

$$\sum_{n=1}^{13} 117 - 7(n-1) = 975$$