2.4 Real Zeros of a Polynomial

Synthetic Division

$(x^3 - x^2 + 2x - 7) \div (2x - 1)$

$2x - 1 = 0$, $+1 \; +1$, $\frac{2x}{2} = \frac{1}{2}$, $x = \frac{1}{2}$

$\frac{1}{2}$ | 1 | −1 | 2 | −7

↓ | | $\frac{1}{2}$ | $-\frac{1}{4}$ | $\frac{7}{8}$

1 | $-\frac{1}{2}$ | $\frac{7}{4}$ | $-\frac{49}{8}$

$1x^2 - \frac{1}{2}x + \frac{7}{4} + \frac{-\frac{49}{8}}{2x-1}$

$x^2 - \frac{1}{2}x + \frac{7}{4} - \frac{\frac{49}{8}}{2x-1}$

Ex $\frac{x^4 - 3x^2 - 2}{x - 5}$

5 | 1 | 0 | −3 | 0 | −2

↓ | | 5 | 25 | 110 | 550

1 | 5 | 22 | 110 | 548

multiply

$1x^3 + 5x^2 + 22x + 110 + \frac{548}{x-5}$

Is $(x-3)$ a factor of $x^3 - x^2 - x - 15$?

(Remainder Theorem) 3 | 1 | −1 | −1 | −15

↓ | | 3 | 6 | 15

1 | 2 | 5 | 0

$1x^2 + 2x + 5$ ☺

(Factor Theorem) $(3)^3 - (3)^2 - (3) - 15 \stackrel{?}{=} 0$

$27 - 9 - 3 - 15 = 0$ ✓

Ex Find all possible rational roots (zeros) and determine if any are zeros. (Rational Roots Theorem)

$2x^4 - 7x^3 - 8x^2 + 14x + 8$

$\pm \frac{\text{factors of constant}}{\text{factors of L.C.}}$

$\pm \frac{1, 2, 4, 8}{1, 2} = \pm 1, \frac{1}{2}, 2, 4, 8$

4 | 2 | −7 | −8 | 14 | 8

↓ | | 8 | 4 | −16 | −8

2 | 1 | −4 | −2 | 0

$(2x^3 + 1x^2)(-4x - 2)$ ◉

$x^2(2x+1) - 2(2x+1)$

$(2x+1)(x^2 - 2)$

$x = -\frac{1}{2}, 4, \pm\sqrt{2}$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$