2.3 Continuity

How would you describe Continuity?

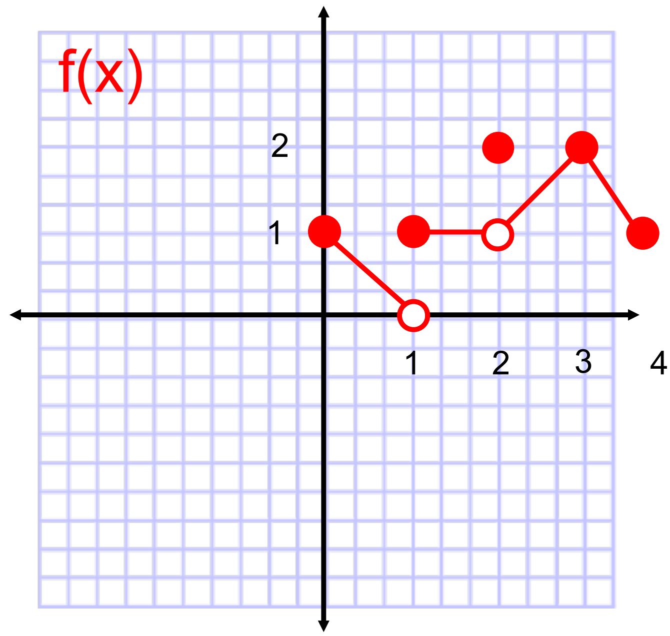
No holes or asymptotes

Where is f(x) continuous?

$[0,1) \cup (1,2) \cup (2,4]$

For what values of c does $\lim_{x \to c} f(x)$ exist?

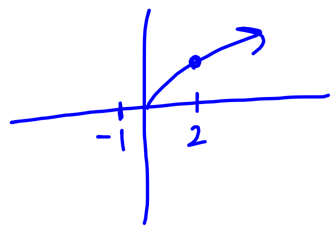
$(0,1) \cup (1,4)$



Where is the function discontinuous?

$x=1 \quad x=2$

Continuity at a point: (RS #23)

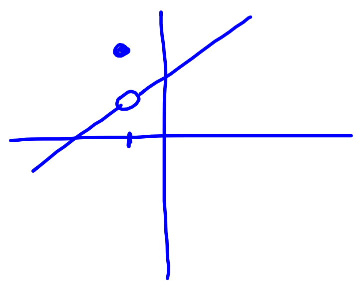


$f(x)$ is continuous at x = c if

1. $f(c)$ exists

2. $\lim\limits_{x \to c} f(x)$ exists (remember this means left hand = right hand)

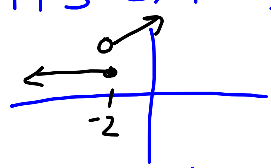
3. $\lim\limits_{x \to c} f(x) = f(c)$



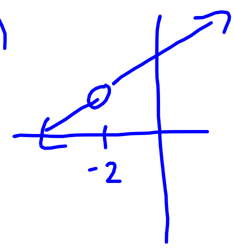
3 types of discontinuities

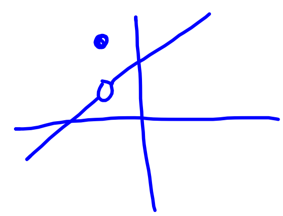
(hole (removable discontinuity))

1. jump: one sided limits exist, but are not equal



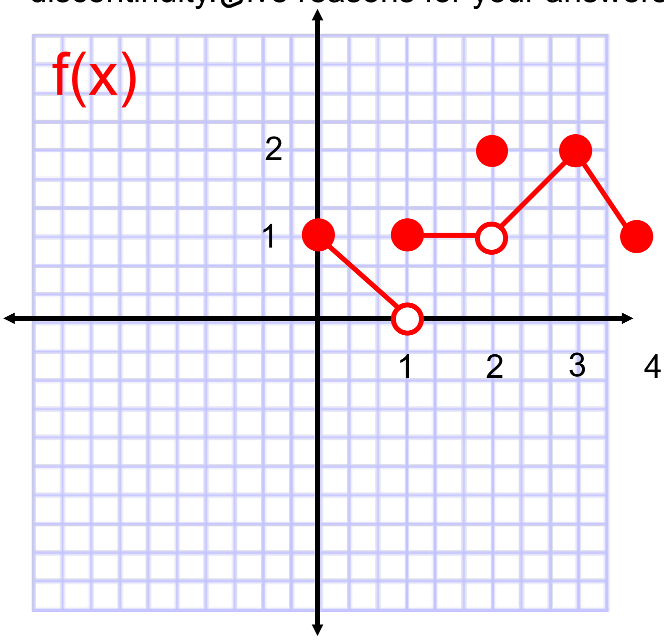
2. removable: the limit at that value exists, but $f(c) \neq \lim_{x \to c} f(x)$





3. infinite: Asymptote

Find the values of x where the graph is not continuous. Identify the type of discontinuity. Give reasons for your answers. (Use the continuity definition to justify)



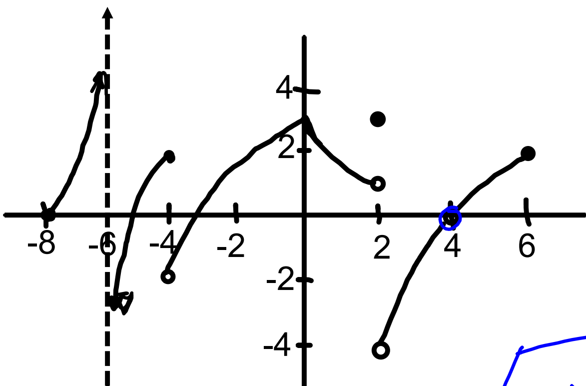
$x = 1$, jump

Reason: $\lim_{x \to 1} f(x)$ DNE

$x = 2$, removable

$f(2) \neq \lim_{x \to 2} f(x)$

Find the values of x where the graph is not continuous. Identify the type of discontinuity. ~~F~~Give reasons for your answers. (Use the continuity definition to justify)



$x = -6$, infinite

① $\lim_{x \to -6} f(x)$ DNE

② $f(-6)$ DNE

$x = -4$, jump, $\lim_{x \to -4} f(x)$ DNE

$x = 2$, jump

$\lim_{x \to 2} f(x)$ DNE

$x = 4$, removable

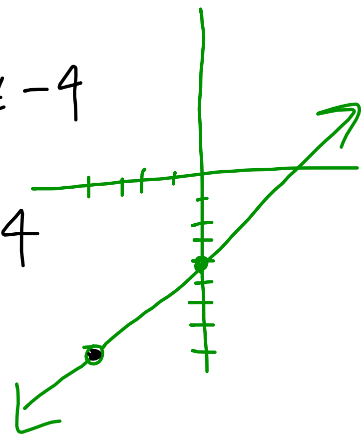
$f(4) \neq \lim_{x \to 4} f(x)$

DNE

Extended function- writing as a piecewise function

Ex. $f(x) = \dfrac{x^2-16}{x+4} = \dfrac{\cancel{(x+4)}(x-4)}{\cancel{x+4}}$

$$f(x) = \begin{cases} \dfrac{x^2-16}{x+4}, & x \neq -4 \\ -8, & x = -4 \end{cases}$$



Continuous function-

A function that is continuous at every point in the domain

Compositions of continuous functions are continuous.

$f(x) = \sin x^2$

$\sin x \quad x^2 \longrightarrow \sin x^2$

$\downarrow \quad \downarrow \quad \downarrow$

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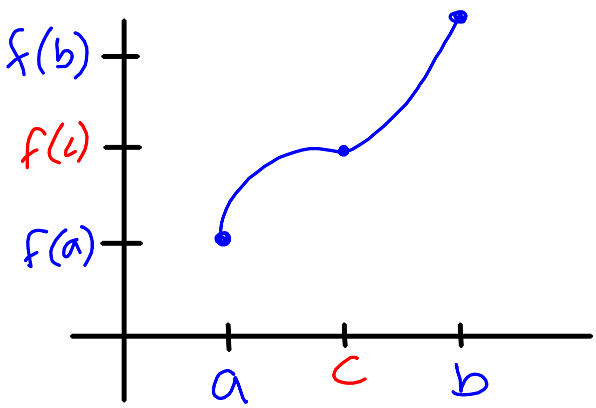
Intermediate Value Theorem (IVT)



If $f(x)$ is continuous over $[a, b]$, then the function takes on every y-value from $f(a)$ to $f(b)$