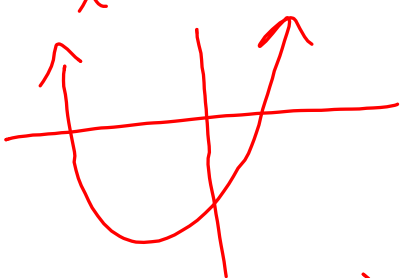
# 2.5 Complex Zeros & Fund. Thm of Alg.

FToA: an nth degree polynomial will have n zeros (could be a combo Real/Imag & some can be repeated)

* odd functions will ALWAYS have at least 1 real zero.
* Imaginary factors come in conjugate pairs.

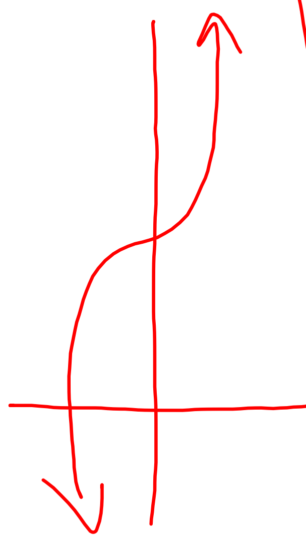
Ex: How many real/imaginary complex are there?

$x^2+5x-7$



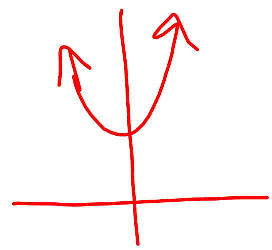
2 zeros (real)

$x^3+8$



3 zeros
(1 real, 2 imag)

$x^2+4$



2 zeros
(both imag)

Write a polynomial of minimum degree with the following zeros:

$x = 4, 7, 2i, -2i$

$f(x) = (x-4)(x-7)(x-2i)(x+2i)$

$x^2 + \cancel{2ix} - \cancel{2ix} - 4i^2$

$+4$

$\boxed{f(x) = (x-4)(x-7)(x^2+4)}$

Ex: Find all zeros and write a linear factorization of

$$x^4 + x^3 + 5x^2 - x - 6$$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 5 | -1 | -6 |
| | | 1 | 2 | 7 | 6 |
| -1 | 1 | 2 | 7 | 6 | ☺ |
| | | -1 | -1 | -6 | |
| | $1x^2$ | $+1x$ | $+6$ | ☺ | |

$$x = \frac{-1 \pm \sqrt{1 - \underbrace{4(1)(6)}_{24}}}{2(1)} = \frac{-1 \pm \sqrt{-23}}{2}$$

$$= \boxed{\frac{-1 \pm i\sqrt{23}}{2}, \pm 1}$$

$$\boxed{f(x) = (x-1)(x+1)\left(x - \left(\frac{-1+i\sqrt{23}}{2}\right)\right)\left(x - \left(\frac{-1-i\sqrt{23}}{2}\right)\right)}$$

Ex: Write the polynomial in standard form.

$f(x) = (x+3i)\cdot(x-3i)$

$x^2 \cancel{-3ix} \cancel{+3ix} \cancel{-9i^2}$

$+9$

$\boxed{f(x) = x^2+9}$

Ex: Write in standard form, given.

$x = -4,\ 2+3i,\ 2-3i$

$f(x) = (x+4)(x-(2+3i))(x-(2-3i))$

$f(x) = (x+4)(x-2-3i)(x-2+3i)$

$(x^2 - 2x \cancel{+3ix} - 2x + 4 \cancel{-6i}$

$\cancel{-3ix} \cancel{+6i} \cancel{-9i^2})$

$+9$

$\boxed{f(x) = (x+4)(x^2-4x+13)}$