

## Section 2.6: Graphs of Rational Functions

**X and Y-intercepts:**

x-intercepts: when  $y=0$  (zeros of numerator)  
 y-intercepts: when  $x=0$

**Ex. 3** Find the x and y intercepts of the following functions:

a.  $f(x) = \frac{x^2 - 2x + 3}{x + 2}$

b.  $f(x) = \frac{3x - 5}{x^2 - 5x + 6}$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(3)}}{2(1)}$$

$$3x - 5 = 0$$

$$+5 \quad +5$$

$$x = \frac{2 \pm \sqrt{-8}}{2}$$

$$\frac{3x}{3} = \frac{5}{3}$$

$$x = \frac{5}{3}$$

NO X-intercepts

$$f(0) = \frac{0 - 0 + 3}{0 + 2} = \frac{3}{2}$$

$$f(0) = \frac{0 - 5}{0 - 0 + 6} = \frac{-5}{6}$$

**End Behavior:**

y-int:  $(0, \frac{3}{2})$

$(0, -\frac{5}{6})$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

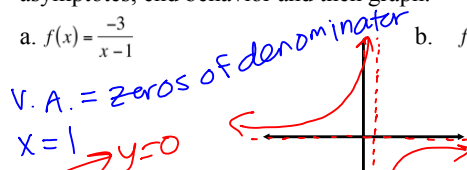
$$\lim_{x \rightarrow \pm\infty} f(x) = 0$$

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

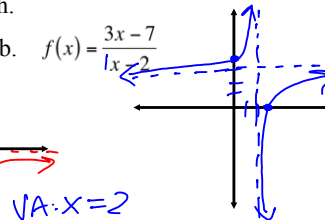
**Ex. 4** Identify the transformations, vertical and horizontal asymptotes, end behavior and then graph.

a.  $f(x) = \frac{-3}{x-1}$

b.  $f(x) = \frac{3x-7}{x-2}$



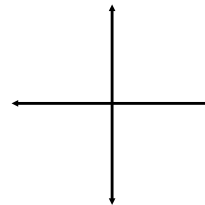
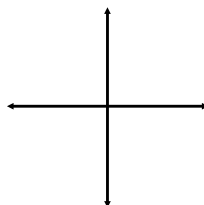
V.A. = zeros of denominator  
 $x=1 \rightarrow y=0$   
 H.A.  $\Rightarrow$  (degree of num/den)  
 \*top heavy  $\rightarrow \frac{x^2}{x}$   
 \*bottom heavy  $\rightarrow \frac{x}{x^2} \rightarrow y=0$   
 \*equal degree  $\rightarrow \frac{Ax}{Bx} \rightarrow y = \frac{A}{B}$



V.A.  $x=2$   
 H.A.  $y=3$   
 $\lim_{x \rightarrow -\infty} f(x) = 3$   
 $\lim_{x \rightarrow \infty} f(x) = 3$

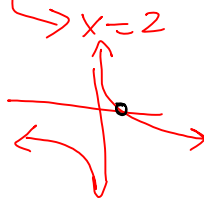
$\lim_{x \rightarrow -\infty} f(x) = 0$   $\lim_{x \rightarrow \infty} f(x) = 0$   
 c.  $f(x) = \frac{3x-2}{x-1}$

d.  $f(x) = \frac{x^2+2}{x^2+1}$



Holes and Oblique/Slant Asymptotes → top heavy equations

$$\frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x-3)}$$

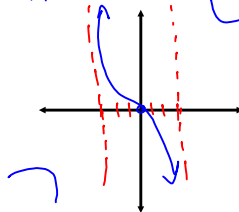


Ex. 5 Find ALL asymptotes, holes and graph.

a.  $f(x) = \frac{x^3}{x^2-9} = \frac{x^3}{(x+3)(x-3)}$

b.  ~~$f(x) = \frac{x^2-9}{x^2-5x+6}$~~

VA:  $x = -3, 3$   
 HA: top heavy → slant

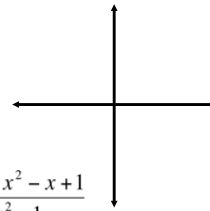


Ex. 6 Find all asymptotes

a.  $f(x) = \frac{x^3 - 3x^2 + 3x + 1}{x-1}$

b.  $f(x) = \frac{2x^5 + x^2 - x + 1}{x^2 - 1}$

VA:  $x = 1$   
 HA: top heavy



Ex. 7 Find the intercepts, all asymptotes, limits at vertical asymptotes, analyze and draw the graph of:

$$f(x) = \frac{x-1}{x^2-x-12} = \frac{x-1}{(x-4)(x+3)}$$

VA:  $x = 4, x = -3$

HA:  $y = 0$

x-int:  $(1, 0)$

y-int:  $(0, 1/12)$

