3.1 Zeros of a Polynomial

Objectives:

- I can find the zeroes of a polynomial by using the factor theorem, remainder theorem, and rational roots theorem

Divide the following polynomials

$x+4\overline{\big)3x^2+7x-20}$

$$\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$$

3-1

Identify the zeros of the following and explain what that means graphically.

$$f(x) = (x+2)(x-1)(x+3)$$

$x = -2, 1, -3$

$x = -2,\ x = 1,\ x = -3$

Write the function in standard form and state the relationship between the degree and zeros of the function

$f(x) = (x+2)(x-1)(x+3)$

$= (x^2 + x - 2)(x+3)$

$= x^3 + x^2 - 2x - 6 + 3x + 3x^2$

$f(x) = x^3 + 4x^2 + x - 6$

* deg: 3

\# of zeros: 3

Remainder Theorem:

For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

Factor Theorem:

If the remainder in $p(x) = (x - a)q(x) + p(a)$ is 0, then $p(x) = (x - a)q(x)$, which tells you that $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then you can write $p(x)$ as $p(x) = (x - a)q(x)$, and when you divide $p(x)$ by $(x - a)$, you get the quotient $q(x)$ with a remainder of 0.

pg. 371

Determine whether the given binomial is a factor of the polynomial p(x). If so, find the remaining factors of p(x).

(B) $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5;\ (x + 1)$

$p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5$

Is $(x+1)$ a factor?

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & -6 & 4 & 5 \\ & \downarrow & -1 & 5 & 1 & -5 \\ \hline & 1 & -5 & -1 & 5 & 0 \end{array}$$

$(1x^3 - 5x^2)(-1x + 5)$

$x^2(x-5) - 1(x-5)$

~~So, $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5 =$~~ [ ].

$(x-5)(x^2-1)$

$p(x) = (x+1)(x-5)(x+1)(x-1)$

$p(x) = (x+1)^2(x-5)(x-1)$

September 20, 2016

pg. 371

Example 3 Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.

(A) $p(x) = x^3 + 3x^2 - 4x - 12; (x + 3)$

pg. 372

Your Turn

Determine whether the given binomial is a factor of the polynomial $p(x)$. If it is, find the remaining factors of $p(x)$.

8. $p(x) = 2x^4 + 8x^3 + 2x + 8; (x + 4)$

9. $p(x) = 3x^3 - 2x + 5; (x - 1)$

Rational Root Theorem:

If all coefficients are integers and the constant is not 0, then all possible rational roots are:

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = 1x^3 + 2x^2 - 19x - 20$$

PR: $\pm \frac{1, 2, 4, 5, 10, 20}{1}$

$x = 4$

$$\begin{array}{r|rrrr} 4 & 1 & 2 & -19 & -20 \\ & \downarrow & 4 & 24 & 20 \\ \hline & 1x^2 & +6x & +5 & 0 \end{array}$$

$(x-4)(x+5)(x+1)$

$x = 4, -5, -1$

⑤ $g(x) = x^4 - 6x^3 + 11x^2 - 6x$

$x(x^3 - 6x^2 + 11x - 6)$

RR: $\pm 1, 2, 3, 6$

1	1	−6	11	−6
	↓	1	−5	6
	$1x^2$	$-5x$	$+6$	☺

$p(x) = x(x-2)(x-3)(x-1)$

$x=0 \quad x-2=0 \quad x-3=0 \quad x-1=0$

$x = 0, 2, 3, 1$

⑧ $f(x) = x^3 - 4x^2 + 2x + 4$

2	1	−4	2	4
+	↓	2	−4	−4
	$1x^2$	$-2x$	-2	☺

$f(x) = (x-2)(x^2 - 2x - 2)$

$x = 2$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{12}}{2} \quad 12 < 3, 4 \quad 4 < 2, 2$$

$$x = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$$

Find all the zeros $f(x) = x^3 - 2x^2 - 8x$

Find all the zeros of: $2x^4 - 7x^3 - 8x^2 + 14x + 8$

Find all the zeros of: $f(x) = x^3 + x^2 - 14x + 6$

Find the polynomial function with a leading coefficient of 2 that has the given degree and zeros: degree 3, zeros -2, 4, 1

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