

## 3-2 Graphing Polynomial Functions

(Book 5.4 pg. 293-306)



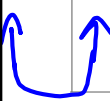
### Objectives:

- I can graph a polynomial function by hand and using technology
- I can find end behavior of a polynomial function
- I can identify zeros, x-intercepts, and factors of a polynomial function
- I can determine the multiplicity of a polynomial function

Graphing Polynomials Task

## End Behavior

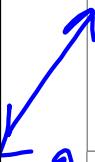


Using a graphing calculator find the end behavior of the following functions. Where do the ends go?

Function	Domain	Range	End Behavior
 $f(x) = x^2$	$(-\infty, \infty)$	$[0, \infty)$	$\lim_{x \rightarrow \infty} f(x)$ As $x \rightarrow +\infty$ , $f(x) \rightarrow \infty$ As $x \rightarrow -\infty$ , $f(x) \rightarrow \infty$
 $f(x) = x^4$	$(-\infty, \infty)$	$[0, \infty)$	As $x \rightarrow +\infty$ , $f(x) \rightarrow \infty$ As $x \rightarrow -\infty$ , $f(x) \rightarrow \infty$
 $f(x) = x^6$	$(-\infty, \infty)$	$[0, \infty)$	As $x \rightarrow +\infty$ , $f(x) \rightarrow \infty$ As $x \rightarrow -\infty$ , $f(x) \rightarrow \infty$

Does it change if I have a negative coefficient? How?

## End Behavior

Using a graphing calculator find the end behavior of the following functions. Where do the ends go?

Function	Domain	Range	End Behavior
 $f(x) = x$	$(-\infty, \infty)$	$(-\infty, \infty)$	As $x \rightarrow +\infty$ , $f(x) \rightarrow \infty$ As $x \rightarrow -\infty$ , $f(x) \rightarrow -\infty$
 $f(x) = x^3$			As $x \rightarrow +\infty$ , $f(x) \rightarrow \infty$ As $x \rightarrow -\infty$ , $f(x) \rightarrow -\infty$
 $f(x) = x^5$			As $x \rightarrow +\infty$ , $f(x) \rightarrow \infty$ As $x \rightarrow -\infty$ , $f(x) \rightarrow -\infty$

Does it change if I have a negative coefficient? How?

## Zeros, x-intercepts, and factors

Find the factors of  $f(x) = x^2 + 4x + 3$   
 $f(x) = (x + 1)(x + 3)$

Now find the x-intercepts of  $f(x) = x^2 + 4x + 3$   
 $(-1, 0)(-3, 0)$

Lastly find the zeros of  $f(x) = x^2 + 4x + 3$   
 $x = -1, -3$

What is the same between the factors, x-intercepts, and zeros of this function?

## Multiplicity

The **power** of the factor determines the nature of the intersection at the point  $x = a$ .  
 (This is referred to as the **multiplicity**.)

### Straight intersection:

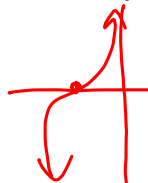
$(x - a)^1$  The power of the zero is 1.

### Tangent intersection : (bounce)

$(x - a)^{\text{even}}$  The power of the zero is even.

### Inflection intersection: (like a slide through)

$(x - a)^{\text{odd}}$  The power of the zero is odd.

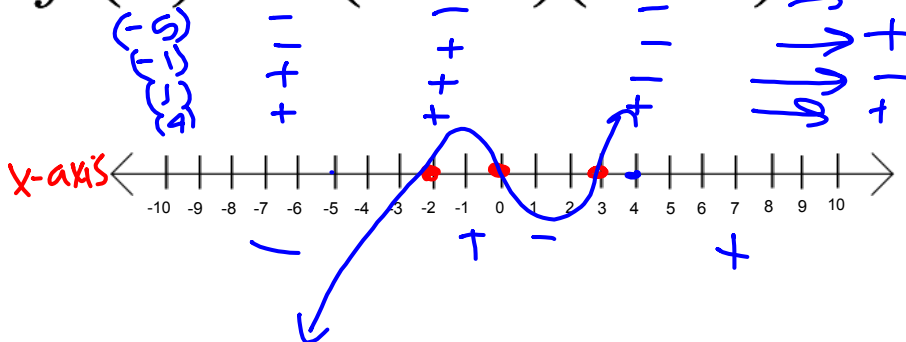


A Use a graphing calculator to graph the cubic functions  $f(x) = x^3$ ,  $f(x) = x^2(x - 2)$ , and  $f(x) = x(x - 2)(x + 2)$ . Then use the graph of each function to answer the questions in the table.

Function	$f(x) = x^3$	$f(x) = x^2(x - 2)$	$f(x) = x(x - 2)(x + 2)$
How many distinct factors does $f(x)$ have?	1	2	3
What are the graph's x-intercepts?	$(0,0)$	$(0,0)(2,0)$	$(0,0)(2,0)(-2,0)$
Is the graph tangent to the x-axis or does it cross the x-axis at each x-intercept?	crosses @ $(0,0)$	tangent/crosses @ 0 / @ 2	crosses @ 3 int
How many turning points does the graph have?	0	2	2
How many global maximum values? How many local?	None	No global 1 local	No global 1 local
How many global minimum values? How many local?	None	"	"

### Sign Chart

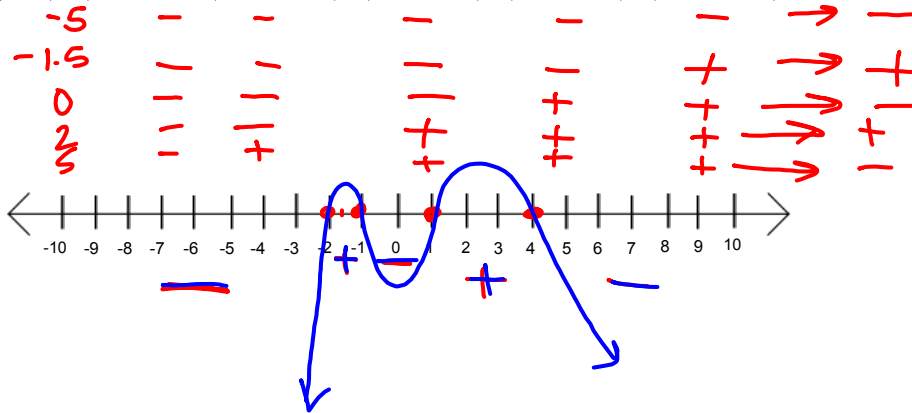
$$f(x) = x(x + 2)(x - 3)$$



Sign Chart

zeros: 4, 1, -1, -2

$$f(x) = -(x - 4)(x - 1)(x + 1)(x + 2)$$



Graphing a Polynomial from factors

③  $f(x) = -(x - 4)(x - 1)(x + 1)(x + 2)$

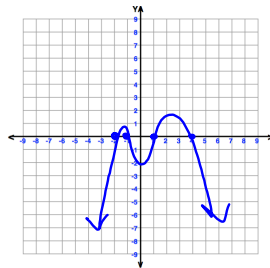
Identify the end behavior.

As  $x \rightarrow +\infty$ ,  $f(x) \rightarrow -\infty$

As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$

Identify the graph's x-intercepts, and then use the sign of  $f(x)$  on intervals determined by the x-intercepts to find where the graph is above the x-axis and where it's below the x-axis.

The x-intercepts are  $x = 4$ ,  $x = 1$ ,  $x = -1$ ,  $x = -2$

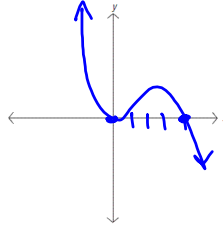


Ex. 8 Find the zeros, the multiplicity, end behavior and graph the following:

a.  $f(x) = -x^2(x-4)$

$x=0, m 2$

$x=4, m 1$

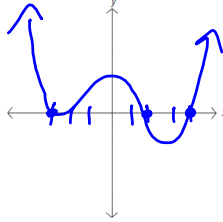


b.  $f(x) = (x+3)^2(x-2)^3(x-4)$

$x = -3, m 2$  (tangent)

$x = 2, m 3$  (inflection)

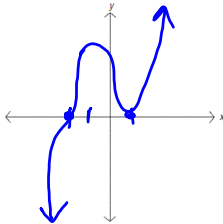
$x = 4, m 1$  (cross)



c.  $f(x) = (x+2)^3(x-1)^2$

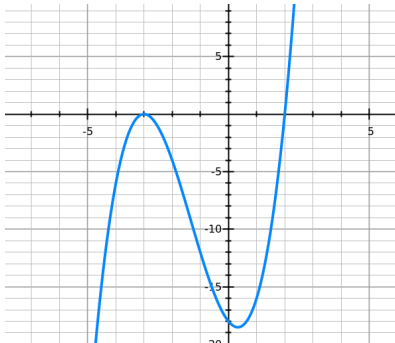
$x = -2, m 3$

$x = 1, m 2$



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Write a function in intercept form for the given graphs whose intercepts are integers. Assume the constant factor of a is either 1 or -1.



$f(x) = (x+3)^2(x-2)$



$f(x) = (x+4)(x+1)^2(x-1)$