3-3 Solving Exponential and Logarithmic equations

Solving Graphically

$$275e^{0.06x} = 1000$$

$y_1 =$ $y_2 =$

Solving Equations Algebraically

-simplify any terms possible without using logarithms

- re-write in logarithmic/exponential form

- use the property of equality for logarithmic equations

Inverses

Addition/Subtraction	Natural Log/e	Common Log/10
$x-5=10$	$e^x=5$	$10^x=100$
$x+7=21$	$\ln x=7$	$\log x=3$

Solve the following equations

A) $10 = 5e^{4x}$

$\frac{10}{5} = \frac{5e^{4x}}{5}$

$2 = e^{4x}$

$\frac{4x}{4} = \frac{\ln 2}{4}$

$x = .17$

B) $5^x - 4 = 7$

$+4 \quad +4$

$5^x = 11$

$x = \log_5 11$

$x = 1.49$

C) $2e^{x-1} + 5 = 80$

$-5 \quad -5$

$\frac{2e^{x-1}}{2} = \frac{75}{2}$

$e^{x-1} = 37.5$

$x - 1 = \ln(37.5)$

$+1$

$x = 4.62$

D) $\frac{20}{20}\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{5}{20}$

$\left(\frac{1}{2}\right)^{x/3} = \frac{1}{4}$

$3 \cdot \frac{x}{3} = (\log_{1/2} \frac{1}{4}) 3$

$x = 6$

How long will it take to triple a $250 initial investment in an account that pays 4.5% compounded quarterly?

$750 = A$ P

$r = 0.045$ $n = 4$

$A = Pe^{rt}$

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$\frac{750}{250} = \frac{250\left(1 + \frac{.045}{4}\right)^{4t}}{250}$$

$$3 = \left(1 + \frac{.045}{4}\right)^{4t}$$

$$\frac{4t}{4} = \frac{\log_{\left(1 + \frac{.045}{4}\right)} 3}{4}$$

$t = 24.6$ yrs

Solve the following

$\log(x) = \log(y)$
$x = y$

A) $\ln(x+12) = 3\ln 2$

$\ln(x+12) = \ln(2^3)$

$x + 12 = 8$
$-12 \quad -12$

$x = -4$

B) $\log x^4 = 2$

$10^2 = x^4$

$\sqrt[4]{100} = \sqrt[4]{x^4}$

$x = 3.2$

C) $4\ln(x+7) - 5 = 1$
$+5 \quad +5$

$\frac{4\ln(x+7)}{4} = \frac{6}{4}$

$\ln(x+7) = 1.5$

$e^{1.5} = x + 7$
$-7 \quad -7$

$x = -2.52$

D) $3 - \log(x+2) = 5$
$-3 \quad -3$

$\frac{-\log(x+2)}{-1} = \frac{2}{-1}$

$\log(x+2) = -2$

$10^{-2} = x + 2$
$-2 \quad -2$

$x = -1.99$

Solve the following

A) $\frac{1}{2}\ln\left(x+3\right)-\ln x=0$

B) $\log(x-2)+\log(x+7)=3\log 4$

Comparing Earthquake intensities:

On the Richter scale, the magnitude M of an earthquake depends on the amount of energy, E (measured in ergs), released by the earthquake as follows:

$$M = \frac{2}{3}\log\frac{E}{10^{11.8}}$$

How much energy is released in a: 7.4 quake compared to a 5.5 quake?

$\frac{3}{2}(7.4) = \frac{3}{2}\log\frac{E}{10^{11.8}}$

$11.1 = \log\frac{E}{10^{11.8}}$

$11.1 = \log E - \log_{10} 10^{11.8}$

$11.1 = \log E - 11.8$

$+11.8 \qquad +11.8$

$22.9 = \log E$

$10^{22.9} = E$

$7.9 \times 10^{22} = E$

$\frac{3}{2}(5.5) = \frac{3}{2}\log\frac{E}{10^{11.8}}$

$8.25 = \log\frac{E}{10^{11.8}}$

$(10^{11.8})(10^{8.25}) = \frac{E}{10^{11.8}} \cdot 10^{11.8}$

1.1×10^{20}

Comparing acidity: $pH = -\log\left[H^+\right]$

H^+ hydrogen-ion concentration

Sour Vinegar has a pH of 2.4 and a box of Leg and Sickle baking soda has a pH of 8.4.

a) what are their hydrogen-ion concentrations

b) how many times greater is the [H+] of vinegar than baking soda?

c) By how many orders of magnitude do they differ?

Newton's Law of Cooling

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

This law states that the temperature difference between an object (*T*) and its surroundings (T_s) decreases exponentially as a function of time (t). Where T_0 is the initial temperature of the object, and *-k* is our constant of variation representing the constant rate of decrease in the temperature difference.

A cup of cocoa has cooled from 95° to 50° after 13 minutes in a room at 25°. How long will it take for the cup to cool to 30°?

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