October 31, 2017

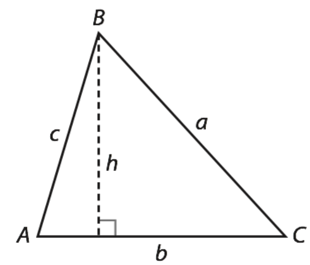
4-3 Law of Sines

Explore Use an Area Formula to Derive the Law of Sines

Recall that the area of a triangle can be found using the sine of one of the angles.

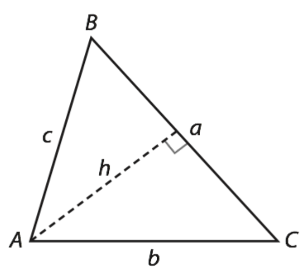
$\text{Area} = \frac{1}{2} b \cdot c \cdot \sin(A)$

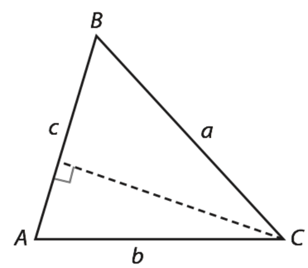
You can write variations of this formula using different angles and sides from the same triangle.



(A) Rewrite the area formula using side length a as the base of the triangle and $\angle C$.

(B) Rewrite the area formula using side length c as the base of the triangle and $\angle B$.





(C) What do all three formulas have in common?

(D) Why is this statement true?

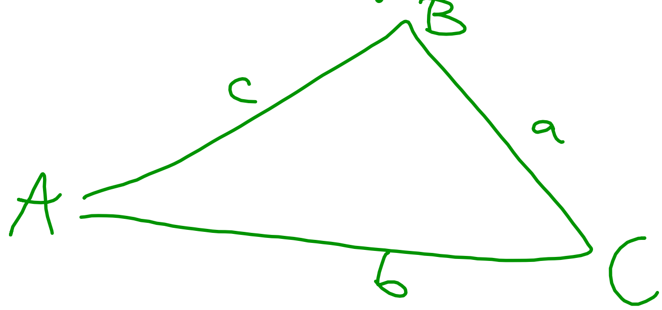
$\frac{1}{2} b \cdot c \cdot \sin(A) = \frac{1}{2} a \cdot b \cdot \sin(C) = \frac{1}{2} c \cdot a \cdot \sin(B)$

(E) Multiply each area by the expression $\frac{2}{abc}$. Write an equivalent statement.

$$\frac{\cancel{2}}{\cancel{a}\cancel{b}c}\left(\frac{1}{\cancel{2}}\cancel{a}\cancel{b}\sin C\right) = \frac{\cancel{2}}{a\cancel{b}\cancel{c}}\left(\frac{1}{\cancel{2}}\cancel{b}\cancel{c}\sin A\right) = \frac{\cancel{2}}{\cancel{a}b\cancel{c}}\left(\frac{1}{\cancel{2}}\cancel{a}\cancel{c}\sin B\right)$$

$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$

Law of Sines

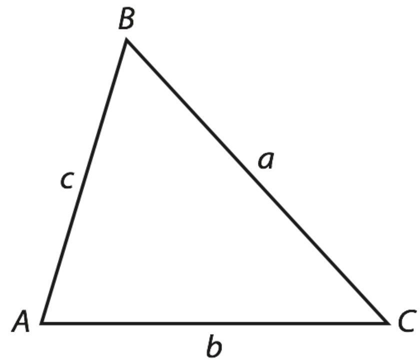


Law of Sines

Law of Sines

Given: $\triangle ABC$

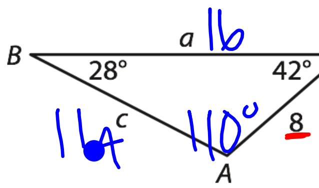
$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$



Your Turn

Find all the unknown measures using the given triangle. Round to the nearest tenth.

4.



$$\frac{\sin 28^\circ}{8} = \frac{\sin 110^\circ}{a}$$

$$\frac{a \cdot \cancel{\sin 28^\circ}}{\cancel{\sin 28^\circ}} = \frac{8 \sin 110^\circ}{\sin 28^\circ}$$

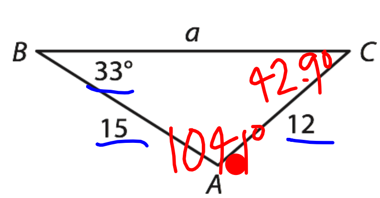
$$\boxed{a = 16}$$

$$\frac{\sin 28^\circ}{8} = \frac{\sin 42^\circ}{c}$$

$$\frac{c \cdot \cancel{\sin 28^\circ}}{\cancel{\sin 28^\circ}} = \frac{8 \sin 42^\circ}{\sin 28^\circ}$$

$$\boxed{c = 11.4}$$

5.



$$\frac{\sin 33^\circ}{12} = \frac{\sin C}{15}$$

$$\frac{15 \sin 33^\circ}{12} = \frac{\cancel{12} \sin C}{\cancel{12}}$$

$$\sin^{-1}\left(\frac{15 \sin 33^\circ}{12}\right) = \cancel{\sin^{-1}}(\sin) C$$

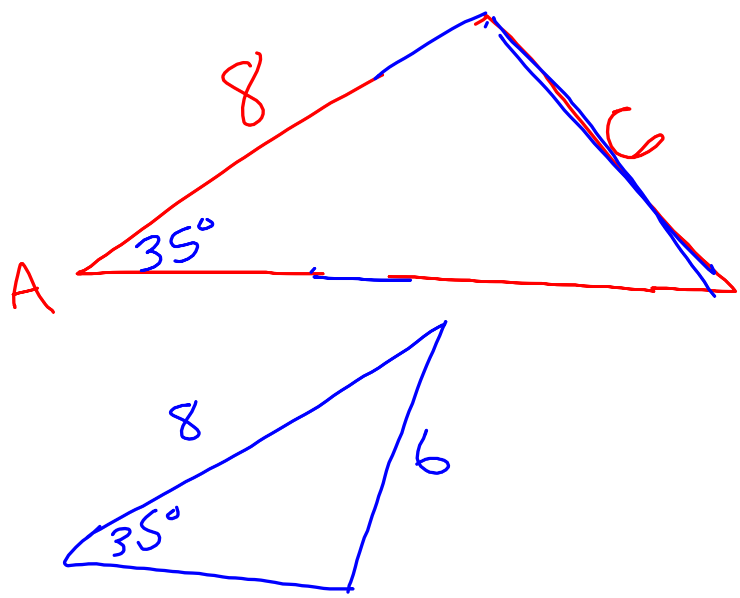
$$\sin^{-1}\left(\frac{15 \sin 33^\circ}{12}\right) = C$$

$$C = 42.9^\circ$$

$$\frac{\sin 33^\circ}{12} = \frac{\sin 104.1}{a}$$

$$\frac{a \cancel{\sin 33^\circ}}{\cancel{\sin 33^\circ}} = \frac{12 \sin 104.1^\circ}{\sin 33^\circ}$$

$$\boxed{a = 21.4}$$


8
6
35°
A
8
6
35°

Solve the triangle given:
$A = 76.7^{o}$
$B = 29.3^{o}$
$c = 87$

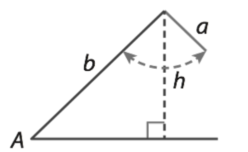
Explain 2 Evaluating Triangles When SSA is Known Information

When you use the Law of Sines to solve a triangle for which you know side-side-angle (SSA) information, zero, one, or two triangles may be possible. For this reason, SSA is called the ambiguous case.

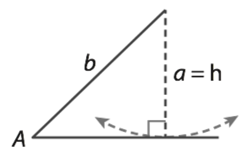
Ambiguous Case

Given a, b, and m$\angle A$.

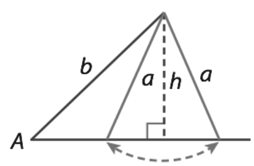
$\angle A$ is acute.



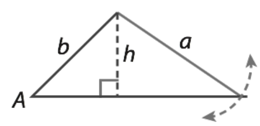
$a < h$
No triangle



$a = h$
One triangle

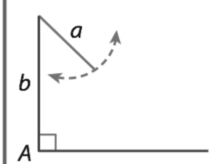


$h < a < b$
Two triangles

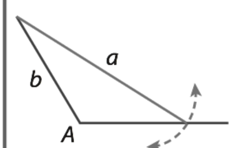


$a \geq b$
One triangle

$\angle A$ is right or obtuse.



$a \leq b$
No triangle



$a > b$
One triangle

Solve the triangle ABC.

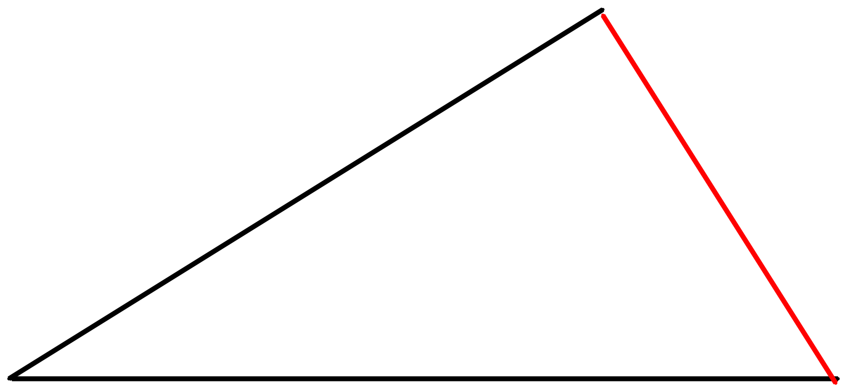
Given $a=20$, $b=5$, $B=42^{o}$

Solve the triangle ABC.

Given: $a=3$, $b=2$, $A=40^{o}$

Solve the triangle ABC.

Given: $a=6$, $b=8$, $A=35^{o}$



Solve the triangle ABC.

Given: $a=37$, $b=40$, $A=71^{o}$