

Finding Complex Zeros and Writing Polynomials from all Zeros

Find all zeros of $p(x)$. Include any multiplicities greater than 1.

1. $p(x) = 3x^3 - 10x^2 + 10x - 4$

$$\begin{array}{r} 2 \overline{) 3 \quad -10 \quad 10 \quad -4} \\ \underline{ 6 \quad -8 \quad 4} \\ 2 \quad 2 \quad 0 \end{array}$$

$(x-2)(3x^2 - 4x + 2)$

$x = \frac{4 \pm \sqrt{16 - 4(3)(2)}}{6}$

$x = 2, 2 \pm \sqrt{2}i$

2. $p(x) = x^3 - 3x^2 + 4x - 12$

$$\begin{array}{r} 3 \overline{) 1 \quad -3 \quad 4 \quad -12} \\ \underline{ 3 \quad -4 \quad 12} \\ 0 \quad 8 \quad 0 \end{array}$$

$(x-3)(x^2 + 4)$

$x = 3, \pm 2i$

Solve the polynomial equation by finding all roots.

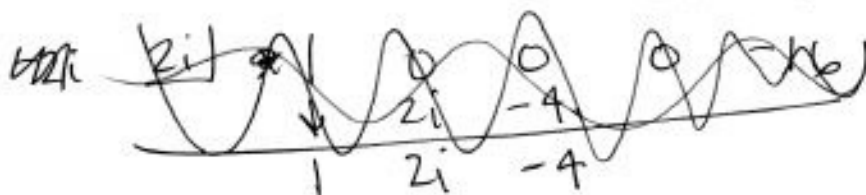
3. $2x^3 - 3x^2 + 8x - 12 = 0$

4. $x^4 - 5x^3 + 3x^2 + x = 0$

$x = \frac{3}{2}, \pm 2i$

$x = 0, 1, 2 \pm \sqrt{5}$

Given the zero $2i$ find the remaining zeros for the polynomial $f(x) = x^4 - 16$



$(x^2 + 4)(x^2 - 4)$
 $x = \pm 2, \pm 2i$

HW 4-3

Name: Key

Write the polynomial function with least degree and a leading coefficient of 1 that has the given zeros.

5. 0, $\sqrt{5}$, and 2

$$f(x) = x(x-2)(x-\sqrt{5})$$

6. $4i$, 2, and -2

$$f(x) = (x-4i)(x+4i)(x+2)(x-2)$$

or

$$f(x) = (x^2+16)(x+2)(x-2)$$

7. 1, -1 (multiplicity 3), and $3i$

$$f(x) = (x-1)(x+1)^3(x-3i)(x+3i)$$

or

$$f(x) = (x-1)(x+1)^3(x^2+9)$$

8. 3 (multiplicity of 2) and $3i$

$$f(x) = (x-3)^2(x-3i)(x+3i)$$

or

$$f(x) = (x-3)^2(x^2+9)$$

9. 2 and $3-2i$

$$f(x) = (x-2)(x-(3-2i))(x+(3-2i))$$
$$= (x-2)(x-3+2i)(x+3-2i)$$

12. Match the roots with their equation.

A. 1

B. -2

C. 2

D. -1

E. $2i$

F. $-2i$

A, B, E, F $x^4 + x^3 + 2x^2 + 4x - 8 = 0$

A, B, C, D $x^4 - 5x^2 + 4 = 0$

Do not do

14. **Explain the Error** A student is asked to write the polynomial function with least degree and a leading coefficient of 1 that has the zeros $1 + i$, $1 - i$, $\sqrt{2}$, and -3 . The student writes the product of factors shown, and multiplies them together to obtain $p(x) = x^4 + (1 - \sqrt{2})x^3 - (4 + \sqrt{2})x^2 + (6 + 4\sqrt{2})x - 6\sqrt{2}$. What error did the student make? What is the correct function?

15. **Critical Thinking** What is the least degree of a polynomial equation that has $3i$ as a root with a multiplicity of 3, and $2 - \sqrt{3}$ as a root with multiplicity 2? Explain.

$$(x - 3i)^3 (x + 3i)^3 (x - (2 - \sqrt{3}))^2$$

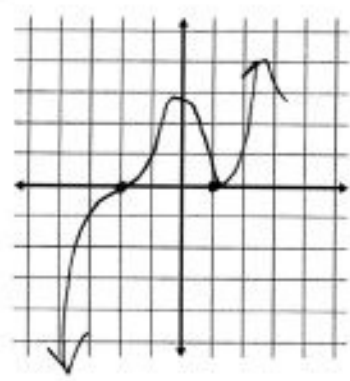
deg: 8

Review

1. Graph the following functions and state the domain, range, and end behavior

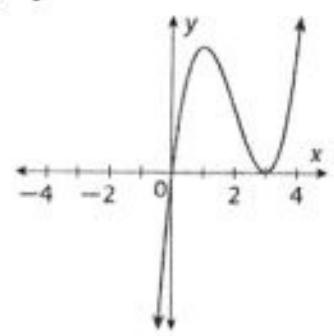
$$f(x) = (x+2)^3(x-1)^2$$

Domain: $(-\infty, \infty)$
 Range: $(-\infty, \infty)$
 End Behavior:



$\lim_{x \rightarrow -\infty} f(x) = -\infty$
 $\lim_{x \rightarrow \infty} f(x) = \infty$

2. Write a function for the following graph



$$f(x) = x(x-3)^2$$