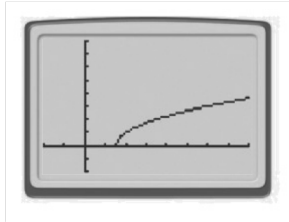


## 5-3 Solving Radical Equations

Remember that you can graph the two sides of an equation as separate functions to find solutions of the equation: a solution is any  $x$ -value where the two graphs intersect.

The graph of  $y = \sqrt{x-3}$  is shown on a calculator window of  $-4 \leq x \leq 16$  and  $-2 \leq y \leq 8$ . Reproduce the graph on your calculator. Then add the graph of  $y = 2$ .



How many solutions does the equation  $\sqrt{x-3} = 2$  have? \_\_\_\_\_ How do you know?

On your calculator, replace the graph of  $y = 2$  with the graph of  $y = -1$ .

How many solutions does the equation  $\sqrt{x-3} = -1$  have? \_\_\_\_\_ How do you know?

Find the solution graphically

$$(4+5)^{\frac{1}{2}} - 2 = 1 \checkmark$$

$$(x+5)^{\frac{1}{2}} - 2 = 1$$

$$\sqrt{x+5} - 2 = 1$$

$$\begin{array}{r} \phantom{\sqrt{x+5}} - 2 = 1 \\ \phantom{\sqrt{x+5}} + 2 \phantom{=} + 2 \end{array}$$

$$(\sqrt{x+5})^2 = (3)^2$$

$$x+5 = 9$$

$$x-5 = -5$$

$$x = 4$$

$$(x+6)^{\frac{1}{2}} - (2x-4)^{\frac{1}{2}} = 0$$

$$\sqrt{x+6} - \sqrt{2x-4} = 0$$

$$\begin{array}{r} \sqrt{x+6} - \sqrt{2x-4} = 0 \\ \phantom{\sqrt{x+6}} + \sqrt{2x-4} \phantom{=} + \sqrt{2x-4} \end{array}$$

$$(\sqrt{x+6})^2 = (\sqrt{2x-4})^2$$

$$x+6 = 2x-4$$

$$\begin{array}{r} x+6 = 2x-4 \\ -x \phantom{=} -x \end{array}$$

$$6 = x-4$$

$$\begin{array}{r} \phantom{6} + 4 \phantom{=} + 4 \end{array}$$

$$x = 10$$

$$2 + \sqrt{-1+10} = -1$$

$$2 + \sqrt{9} = -1$$

$$2 + 3 \neq -1$$

$$2 + \sqrt{x+10} = x$$

$$\begin{array}{r} \phantom{2 + \sqrt{x+10}} - 2 \phantom{=} - 2 \end{array}$$

$$(\sqrt{x+10})^2 = (x-2)^2$$

$$\begin{array}{r} (x+10) = (x-2)(x-2) \\ x+10 = x^2 - 4x + 4 \\ -x - 10 \phantom{=} -x - 10 \\ 0 = x^2 - 5x - 6 \\ 0 = (x-6)(x+1) \\ x = 6, -1 \end{array}$$

Solve the following, check for extraneous solutions

$$(2\sqrt{x})^2 = (3\sqrt{x-2})^2$$

$$4x = 9(x-2)$$

$$4x = 9x - 18$$

$$-9x = -18$$

$$x = \frac{18}{9}$$

$$x = \frac{18}{9}$$

$$\sqrt{5x-11} = x-1$$

$$x = 3, 4$$

$$\sqrt{2x+5} + 4 = 3$$

No solution  $\emptyset$

$$\sqrt{2(-2)+5} + 4 = 3$$

$$\sqrt{1} + 4 = 3$$

$$1 + 4 \neq 3$$

**Example 2** Solve the equation.

$$\sqrt[3]{x+2} + 7 = 5$$

$$(\sqrt[3]{x+2})^3 = (-2)^3$$

$$x+2 = -8$$

$$x = -10$$

Solve the following:

$$(\sqrt[3]{x-5})^3 = (\sqrt[3]{7-x})^3$$

$$x-5 = 7-x$$

$$2x = 12$$

$$x = 6$$

$$2(x-50)^{\frac{1}{3}} = -10$$

$$\frac{2\sqrt[3]{x-50}}{2} = \frac{-10}{2}$$

$$(\sqrt[3]{x-50})^3 = (-5)^3$$

$$x-50 = -125$$

$$x = -75$$

$$(\sqrt[3]{x+2})^3 = (\sqrt[3]{x+3})^3$$

$$x+2 = x+3$$

$$2 \neq 3$$

NO SOLUTION