5.1 Fundamental Identities

Identity:

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equality that is true for all values of the domain for both expressions as long as they are both defined

$$\tan\theta \bullet \cos\theta = \sin\theta$$

this is true for all θ, as long as sinθ, cosθ, and tanθ are defined

Reciprocal & Quotient Relationships

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

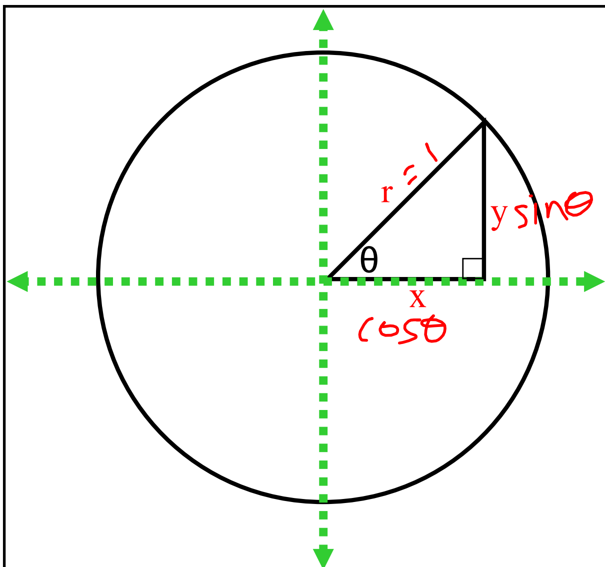
$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$\tan\theta\cos\theta = \sin\theta$

$\dfrac{\sin\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta} = \sin\theta$ ✓



Can we write an equation about the relationship of x, y, and z?

$\sin^2\theta + \cos^2\theta = 1$

Pythagorean Relationships

$$x^2 + y^2 = r^2$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

Pythagorean Relationships

$$1+\tan^2\theta=\sec^2\theta$$

$$1=\sec^2\theta-\tan^2\theta$$

$$\tan^2\theta=\sec^2\theta-1$$

$$\cancel{\sin^2\theta+\cos^2\theta=1}$$

$$1+\cot^2\theta=\csc^2\theta$$

$$1=\csc^2\theta-\cot^2\theta$$

$$\cot^2\theta=\csc^2\theta-1$$

Pythagorean Relationships

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$y = \sin\left(x + \frac{\pi}{2}\right)$$

Co-Function Identities

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta \qquad \csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$$

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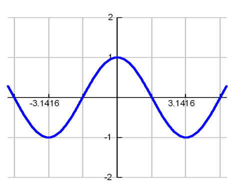
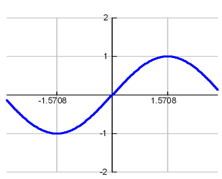
Odd/Even Identities



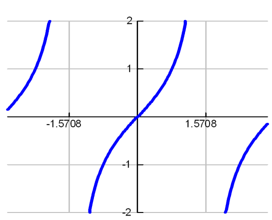
$$\cos(-x)=\cos x$$

$$\sec(-x)=\sec x$$



$$\sin(-x)=-\sin x$$

$$\csc(-x)=-\csc x$$



$$\tan(-x)=-\tan x$$

$$\cot(-x)=-\cot x$$

Simplify:

$\sin x \csc(-x) = \sin x(-\csc x) = -\frac{\cancel{\sin x}}{1}\left(\frac{1}{\cancel{\sin x}}\right)$

$= \boxed{-1}$

$\cot x \tan x$

$\frac{\cancel{\cot x}}{1} \cdot \frac{1}{\cancel{\cot x}} = \boxed{1}$

Perfect Squares:

$x^2 - 8x + 16$

$(x-4)^2$

$x^2 + 14x + 49$

$(x+7)^2$

$\sin^2 x - 10\sin x + 25$

$(\sin x - 5)^2$

$\cos^2 x + 16\cos x + 64$

$(\cos x + 8)^2$

Difference of Squares:

$x^2 - 16$

$(x+4)(x-4)$

$x^2 - 49$

$(x+7)(x-7)$

$1 - x^2$

$(1-x)(1+x)$

$1 - \sin^2 x$

$(1+\sin x)(1-\sin x)$

$\sin^2 x - \cos^2 x$

$(\sin x + \cos x)(\sin x - \cos x)$

Simplify:

$$\frac{1-\cos^2 x}{1+\cos x} = \frac{(1-\cos x)\cancel{(1+\cos x)}}{\cancel{1+\cos x}} = \boxed{1-\cos x}$$

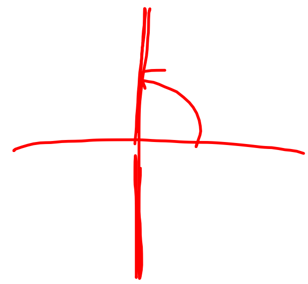
$$\frac{1^{(1+\sin x)}}{(1-\sin x)(1+\sin x)} + \frac{1(1-\sin x)}{(1+\sin x)(1-\sin x)}$$

$$\frac{1+\cancel{\sin x} + 1 - \cancel{\sin x}}{1 - \cancel{\sin x} + \cancel{\sin x} - \sin^2 x}$$

$$\frac{2}{1-\sin^2 x} = \frac{2}{\cos^2 x} = \boxed{2\sec^2 x}$$

Solve the Equation for $[0, 2\pi)$

$$\tan x \sin^2 x = \tan x$$



How you write all solutions