5.3 Sum & Difference Identities

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Find the exact value of:

$\cos 105° = \cos(60+45)$

$= \cos 60 \cos 45 - \sin 60 \sin 45$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$\sin 15° = \sin(60-45)$

$= \sin 60 \cos 45 - \cos 60 \sin 45$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\tan\left(\frac{5\pi}{12}\right) = \tan\left(\pi/6 + \pi/4\right)$

$\frac{2\pi}{12} + \frac{3\pi}{12}$

$\frac{\pi}{6} + \frac{\pi}{4}$

$= \frac{\tan \pi/6 + \tan \pi/4}{1 - \tan \pi/6 \tan \pi/4}$

$= \frac{\frac{\sqrt{3}}{3} + \frac{3}{3}}{1 - \left(\frac{\sqrt{3}}{3}\right)(1)} = \frac{\sqrt{3}+3}{3} \div \frac{3-\sqrt{3}}{3}$

$\frac{3}{3} - \frac{\sqrt{3}}{3} = \frac{3-\sqrt{3}}{3}$

$\frac{\sqrt{3}+3}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}}$

$\boxed{\frac{\sqrt{3}+3}{3-\sqrt{3}}}$

Write as the sin, cos, or tan of an angle:

$$\sin 50° \cos 26° - \cos 50° \sin 26°$$

$\sin(50-26) = \boxed{\sin 24°}$

$$\cos 50° \cos 26° - \sin 50° \sin 26°$$

$\cos(50+26) = \boxed{\cos 76°}$

$$\frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \tan\left(\frac{4\pi}{\cancel{3}\,4\,3} - \frac{\pi\,3}{\cancel{4}\,3}\right)$$

$\frac{4\pi}{12} - \frac{3\pi}{12}$

$\boxed{\tan\left(\pi/12\right)}$

Prove the identity:

$$\cos\left(x - \frac{\pi}{2}\right) = \sin x$$

$\cos x \cos \pi/2 + \sin x \sin \pi/2$

$\cos x (0) + \sin x (1)$

$\sin x$ ✓

If one of the angles in a sum or difference is a quadrantal angle (multiple of 90 or $\pi/2$) then the sum-difference identities yield single-termed expressions.

We call these reduction formulas

$$\underbrace{\sin(x-y)}+\underbrace{\sin(x+y)}=2\sin x\cos y$$

$\sin x\cos y \cancel{- \cos x \sin y} + \sin x \cos y \cancel{+ \cos x \sin y}$

$\boxed{2\sin x\cos y}$ ✓

~~Use your knowledge of identities to~~ describe the transformation of these graphs.

$\cos(x-3)$

right 3

$\cos(3-2x)$

$\cos(-2x+3)$

$\cos\left(-2\left(x-\frac{3}{2}\right)\right)$

- right $3/2$
- reflect horizontally
- horiz compression by 2

Just like in 5.1 we can use identities to help us solve equations using trig

$$\cos 3x \cos x = \sin 3x \sin x$$

$$\sin(3x) = 3\cos^2 x \sin x - \sin^3 x$$

$\sin(2x+x) = \sin 2x \cos x + \cos 2x \sin x$

$\sin(x+x)$ $\cos(x+x)$

$(\sin x \cos x + \cos x \sin x)\cos x + (\cos x \cos x - \sin x \sin x)\sin x$

$\sin x \cos^2 x + \cos^2 x \sin x + \cos^2 x \sin x - \sin^3 x$

$\boxed{3\cos^2 x \sin x - \sin^3 x}$ ✓