Double Angle

$$\sin 2x = 2\sin x \cos x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x\end{aligned}$$

5.4 Multiple Angle Identities

Half Angle

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{1-\cos x}{\sin x}$$

$$= \frac{\sin x}{1+\cos x}$$

Use the half angle identities to find the exact value without a calculator

$\sin 15^\circ$

positive

$\sin\left(\frac{30}{2}\right) = \sqrt{\frac{1-\cos 30}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{\frac{2}{2}-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}} \approx .26$

$\sin(45-30)$

Prove: $(\sin x+\cos x)^2 = 1+\sin 2x$

$(\sin x+\cos x)(\sin x+\cos x)$

$\sin^2 x + \sin x\cos x + \sin x\cos x + \cos^2 x$

$1 + 2\sin x\cos x$

$1 + \sin 2x$ ✓

$$\frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

$$\frac{2\tan x}{\sec^2 x} = \frac{2\tan x}{1} \cdot \frac{1}{\sec^2 x}$$

$$= 2\tan x \cos^2 x$$

$$= 2\,\frac{\sin x}{\cos x}\cos^2 x$$

$$= 2\sin x \cos x$$

$$= \sin 2x \checkmark$$

Solve the equation: $[0, 2\pi]$ $0 \le x < 2\pi$ $0 \le x < 360°$

$$\sin 2x + \cos x = 0$$

$$2\sin x\cos x + \cos x = 0$$

$$\cos x(2\sin x + 1) = 0$$

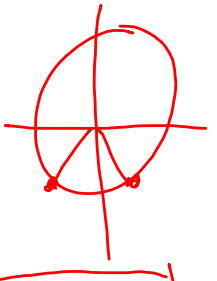
$\cos x = 0$

$90°, 270°$

$2\sin x + 1 = 0$

$$\frac{2\sin x}{2} = \frac{-1}{2}$$

$$\sin x = \frac{-1}{2}$$



$210°, 330°$