6-4 Inverse Functions

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x).

Notation:

$$f^{-1}(x)$$

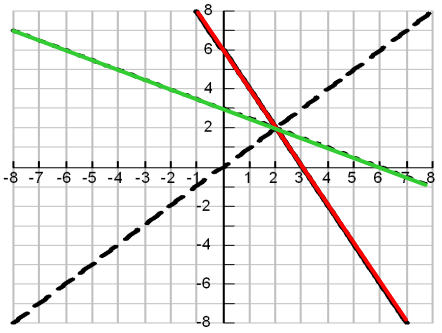
Represents the inverse of the function $f(x)$

Horizontal-Line Test

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.

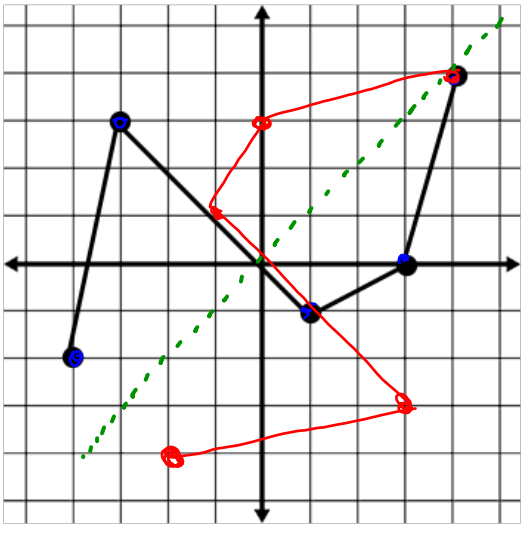
Show $f(x) = 6 - 2x$ and $g(x) = \frac{6 - x}{2}$ are inverses graphically.



f(x): (1,4) (3,0) (4,-2)

g(x): (4,1) (0,3) (-2,4)

Graph the inverse of the graph. (Use y=x to find inverse points)



Inverse

(-4,-2) → (-2,-4)

(-3,3) → (3,-3)

(1,-1) → (-1,1)

(3,0) → (0,3)

(4,4) → (4,4)

To find the inverse equation of a function

1. Change $f(x)$ to y.
2. Interchange x and y
3. Solve for y
4. Change new y to $f^{-1}(x)$

Find the inverse of each function

$f(x) = x^2 + 1$

$y = x^2 + 1$

$x = y^2 + 1$

$-1 \quad -1$

$\pm\sqrt{x-1} = \sqrt{y^2}$

$y = \pm\sqrt{x-1}$

$f^{-1}(x) = \pm\sqrt{x-1}$

$g(x) = \dfrac{x+1}{2x+3}$

y

$x = \dfrac{y+1}{2y+3}$ $(2y+3)$ $2y+3$

$2xy + 3x = y + 1$

$-y \quad -3x \quad -y \quad -3x$

$2xy - y = 1 - 3x$

$\dfrac{y(2x-1)}{2x-1} = \dfrac{1-3x}{2x-1}$

$y = \dfrac{1-3x}{2x-1}$

$g^{-1}(x) = \dfrac{1-3x}{2x-1}$

Find the inverse of each function.

$h(x) = 2x^3 + 3$

$h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$

$g(x) = \sqrt[3]{x} - 3$

$g^{-1}(x) = (x+3)^3$

We can verify that two functions are inverses of each other by determining if the composition of the two functions are both equal to x.

$f \circ g = x$ $\quad f(g(x))$

$g \circ f = x$ $\quad g(f(x))$

$f \circ f^{-1} = x$

$f^{-1} \circ f = x$

Use composition to determine if the following functions are inverses of each other.

$f(x) = 5x + 1$

$g(x) = \frac{x-1}{5}$

$f \circ g = 5\left(\frac{x-1}{5}\right) + 1$

$x - 1 + 1 = x$ ✓

$g \circ f = \frac{(5x+1)-1}{5} = \frac{5x}{5}$

$= x$ ✓