7-1 Sequences

Warm up:

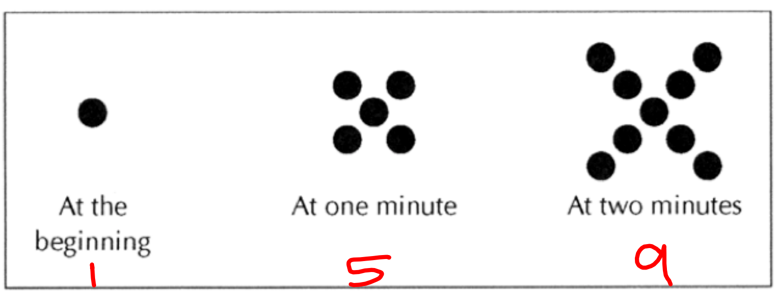
Write the next 3 terms for the following:

1 2 3

a. $\{1,4,9,16,\underline{25},\underline{36},\underline{49}\}$ n^2

b. $\{13,21,29,37,\underline{45},\underline{53},\underline{61}\}$ $+8$

c. $\{15,5,\frac{5}{3},\frac{5}{9},\frac{5}{27},\frac{5}{81},\frac{5}{243}\}$ $\cdot\frac{1}{3}$



1. Describe the pattern that you see in the sequence of figures above.

adding 4

2. Assuming the sequence continues in the same way, how many dots are there at 3 minutes? At 4 minutes?

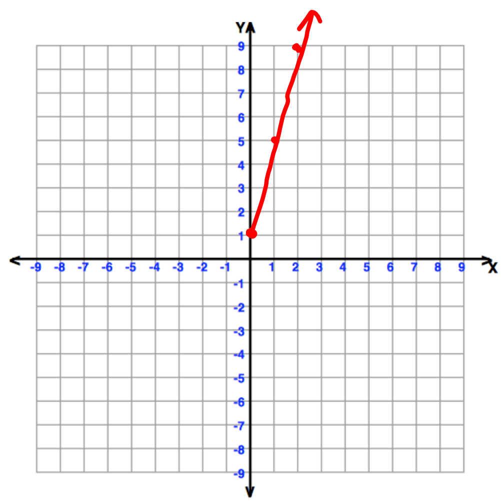
13 17

3. Write an equation to represent the pattern

$y=4x+1$

4. Make a table of values and graph

x	y
0	1
1	5
2	9
3	13



Arithmetic Sequence

arithmetic - sequence with common difference between successive terms (repeated addition)

explicit - each term is defined independently

$$f(n)=a+dn \quad for \quad n\geq 0$$

$$f(n)=a+d(n-1) \quad for \quad n\geq 1$$

recursive - use the previous term to define the following terms

$$f(0)=a, \quad f(n)=f(n-1)+d \quad n\geq 1$$

$$f(1)=a, \quad f(n)=f(n-1)+d \quad n\geq 2$$

a= initial value

d= common Difference

n= term #

Example 1 Use the given table to write an explicit and a recursive rule for the sequence.

(A)

n	0	1	2	3	4	5
$f(n)$	2	5	8	11	14	17

$+3$

$f(0)=2=a$

explicit

$$\boxed{f(n)=2+3n}$$

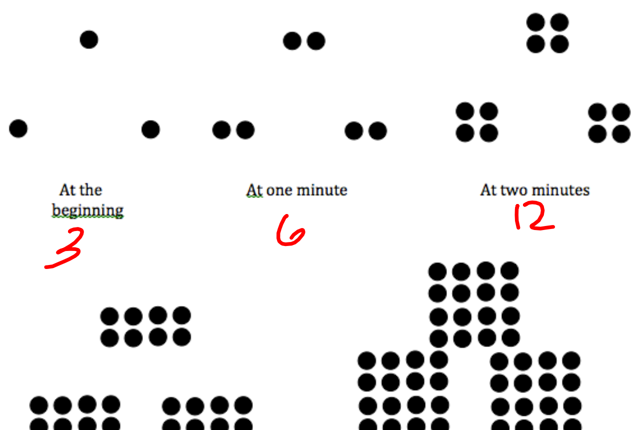
recursive

$f(0)=2$

$f(1)=5$

$$\boxed{f(n)=f(n-1)+3}$$

$f(2)=5+3=8$



At the beginning 3

At one minute 6

At two minutes 12

24 At three minutes

48 At four minutes

1. Describe the pattern that you see in the sequence of figures above. Doubling

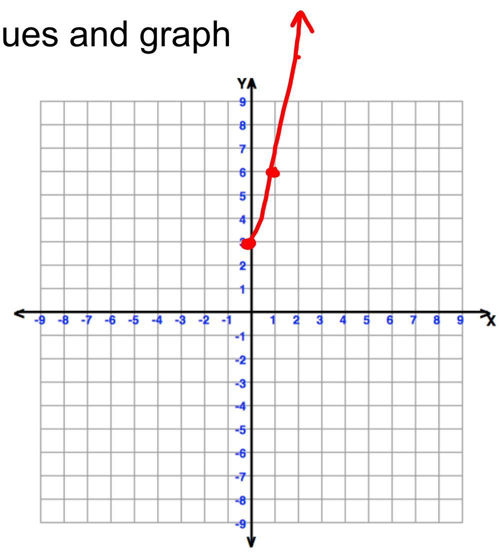
2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes? 96

3. Write an equation to represent the pattern

$f(n) = 3 \cdot 2^n$

4. Make a table of values and graph

x	y
0	3
1	6
2	12
3	24



Geometric Sequence

geometric - sequence with a common factor between successive terms (repeated multiplication)

explicit: $f(n) = ar^{n-1}$

recursive: $f(n) = \underline{r} \bullet f(n-1),\ for\ f(0) = a$

a= initial value

r= common Ratio

n= term #

Write explicit and recursive rules to represent the table

Ⓐ

n	0	1	2	3	4	...	$j-1$	j	...
$f(n)$		6	12	24	48	...	$ar^{(j-1)}$	ar^{j}	...

$\cdot 2$ $\quad \cdot 2$

$f(1) = 6$

$r = 2$

explicit

$f(n) = 6 \cdot 2^{(n-1)}$

recursive

$f(1) = 6$

$f(n) = 2 \cdot f(n-1)$

Write explicit and recursive rules to represent the table

Ⓑ

n	1	2	3	4	5	...	$j-1$	j	...
$f(n)$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	...	$ar^{(j-1)}$	ar^{j}	...

$\cdot 5$

$r = 5$

$f(1) = 1/25$

$f(n) = \frac{1}{25} \cdot 5^{(n-1)}$

recursive

$f(1) = \frac{1}{25}$

$f(n) = 5 \cdot f(n-1)$

Your Turn

Write the explicit and recursive rules for a geometric sequence given a table of values.

4.

n	0	1	2	3	4	5	6	...
$f(n)$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	...

$f(n) = \frac{1}{27} \cdot 3^n$

explicit

$f(0) = 1/27$

$f(n) = f(n-1) \cdot 3$

recursive

5.

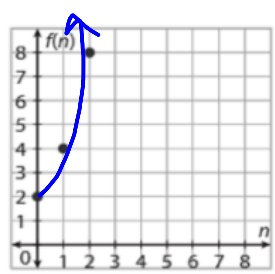
n	1	2	3	4	5	6	7	...
$f(n)$	0.001	0.01	0.1	1	10	100	1000	...

$f(n) = 0.001 \cdot 10^{(n-1)}$

$f(n) = f(n-1) \cdot 10$

$f(1) = 0.001$

Example 2 Given either an explicit or recursive rule for a geometric sequence, use a table to generate values and draw the graph of the sequence.



(A) Explicit rule: $f(n) = 2 \cdot 2^n, n \geq 0$

Use a table to generate points.

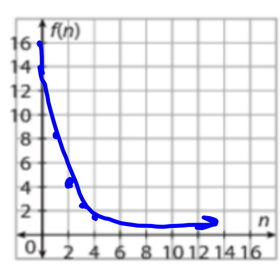
n	0	1	2	3	4	5	...
$f(n)$	2	4	8	16	32	64	...

Plot the first three points on the graph.

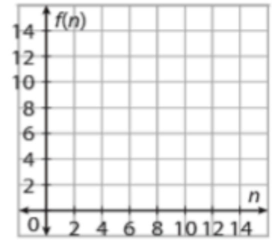
(B) Recursive rule: $f(n) = 0.5 \cdot f(n-1), n \geq 1$ and $f(0) = 16$

Use a table to generate points.

n	0	1	2	3	4	5	6	...
$f(n)$	16	8	4	2	1	½	¼	...

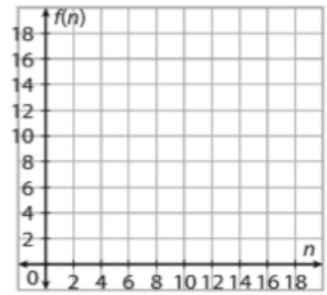


Given either an explicit or recursive rule for a geometric sequence, use a table to generate values and draw the graph of the sequence.



6. $f(n) = 3 \cdot 2^{n-1}, n \geq 1$

n	1	2	3	4	5	...
f(n)						...



7. $f(n) = 3 \cdot f(n-1), n \geq 2$ and $f(1) = 2$

n	1	2	3	4	5	...
f(n)						...

Example 3 **Write both an explicit and recursive rule for the geometric sequence that models the situation. Use the sequence to answer the question asked about the situation.**

(A) The Wimbledon Ladies' Singles Championship begins with 128 players. Each match, two players play and only one moves to the next round. The players compete until there is one winner. How many rounds must the winner play?

Analyze Information

Identify the important information:

- The first round requires 64 matches, so $a = \boxed{64}$.
- The next round requires half as many matches, so $r = \boxed{1/2}$.

Formulate a Plan

Using the fact that the domain starts at 1 and the first round has 128 players, create the explicit rule and the recursive rule for the tournament. The final round will have 1 match(es), so substitute this value into the explicit rule and solve for n.

explicit

$$f(n) = 64 \cdot \frac{1}{2}^{(n-1)}$$

$$\frac{1}{64} = \frac{64 \cdot \frac{1}{2}^{(n-1)}}{64}$$

$$\frac{1}{64} = \frac{1}{2}^{(n-1)}$$

$$\left(\frac{1}{2}\right)^6 = \frac{1}{2}^{n-1}$$

$$6 = n - 1$$
$$+1 \quad +1$$
$$\boxed{7 = n}$$

recursive:

$$f(n) = f(n-1) \cdot \frac{1}{2}$$
$$f(1) = 64$$

Solve

The explicit rule is $f(n) = \boxed{64 \cdot \frac{1}{2}^{n-1}}$, $n = \geq 1$.

The recursive rule is $f(n) = \boxed{\frac{1}{2}} \cdot f(n-1)$, $n \geq 2$ and $f(1) = \boxed{64}$.

The final round will have 1 match, so substitute 1 for $f(n)$ into the explicit rule and solve for n.

$n = 7$

Two powers with the same positive base other than 1 are equal if and only if the exponents are equal.

$a^x = a^y \rightarrow x = y$

The winner must play in 7 rounds.

Justify and Evaluate

The answer of 7 rounds makes sense because using the explicit rule gives $f(7) = \boxed{}$ and the final round will have 1 match(es). This result can be checked using the recursive rule, which again results in $f(7) = \boxed{}$.

Your Turn

Write both an explicit and recursive rule for the geometric sequence that models the situation. Use the sequence to answer the question asked about the situation.

8. A particular type of bacteria divides into two new bacteria every 20 minutes. A scientist growing the bacteria in a laboratory begins with 200 bacteria. How many bacteria are present 4 hours later?

Elaborate

9. Describe the difference between an explicit rule for a geometric sequence and a recursive rule.

10. How would you decide to use $n = 0$ or $n = 1$ as the starting value of n for a geometric sequence modeling a real-world situation?

11. **Essential Question Check-In** How can you define a geometric sequence in an algebraic way? What information do you need to write these rules?