7.1 Zeros of a Polynomial

Divide the following polynomials

$x+4\overline{)3x^2+7x-20}$

$$\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$$

Identify the zeros of the following and explain what that means graphically.

$$f(x)=(x+2)(x-1)(x+3)$$

$0=(x+2)(x-1)(x+3)$

$x+2=0 \quad x-1=0 \quad x+3=0$

$\boxed{x=-2 \quad x=1 \quad x=-3}$

Write the function in standard form and state the relationship between the degree and zeros of the function

$f(x)=(x+2)(x-1)(x+3)$

$(x^2+x-2)(x+3)$

$f(x)=x^3+x^2-2x+3x^2+3x-6$

$\boxed{=x^3+4x^2+x-6}$

deg: 3

Remainder Theorem:

For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$

Factor Theorem:

If the remainder in $p(x) = (x - a)q(x) + p(a)$ is 0, then $p(x) = (x - a)q(x)$, which tells you that $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x - a)$ is a factor of $p(x)$, then you can write $p(x)$ as $p(x) = (x - a)q(x)$, and when you divide $p(x)$ by $(x - a)$, you get the quotient $q(x)$ with a remainder of 0.

pg. 371

(B) $p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5; (x + 1)$

$p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5; (x+1)$

-1	1	-4	-6	4	5
	$\downarrow$	-1	5	1	-5

$(x^3 - 5x^2)(-1x + 5)$ ☺

$x^2(x - 5) - 1(x - 5)$

$(x - 5)(x^2 - 1)$

So, ~~$p(x) = x^4 - 4x^3 - 6x^2 + 4x + 5 =$~~ ________.

$p(x) = (x - 5)(\underline{x + 1})(x - 1)(\underline{x + 1})$

pg. 371

Example 3 Determine whether the given binomial is a factor of the polynomial $p(x)$. If so, find the remaining factors of $p(x)$.

(A) ~~$p(x) = x^3 + 3x^2 - 4x - 12; (x + 3)$~~

Your Turn

Determine whether the given binomial is a factor of the polynomial $p(x)$. If it is, find the remaining factors of $p(x)$.

8. ~~$p(x) = 2x^4 + 8x^3 + 2x + 8; (x + 4)$~~

$p(x) = 3x^3 - 2x + 5; (x - 1)$

$$\begin{array}{r|rrrr} 1 & 3 & 0 & -2 & 5 \\ & \downarrow & 3 & 3 & 1 \\ \hline & 3 & 3 & 1 & 6 \end{array}$$

9. ~~$p(x) = 3x^3 - 2x + 5; (x - 1)$~~

$\boxed{\text{Not a Factor}}$

Rational Root Theorem:

If all coefficients are integers and the constant is not 0, then all possible rational roots are:

$$x = \pm \frac{\text{factors of constant}}{\text{factors of leading coefficient}}$$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$f(x) = x^3 + 2x^2 - 19x - 20$

$x = \pm \frac{1, 2, 4, 5, 10, 20}{}$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -19 & -20 \\ & \downarrow & 1 & 3 & -16 \\ \hline & 1 & 3 & -16 & \cancel{36} \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -19 & -20 \\ & \downarrow & -1 & -1 & 20 \\ \hline & 1x^2 & +1x & -20 & \boxed{☺} \end{array}$$

$(x^2 + x - 20)$

$f(x) = (x+5)(x-4)(x+1)$

Find the rational zeros of the polynomial function; then write the function as a product of factors.

$$f(x)=x^4-4x^3-7x^2+22x+24$$

$$f(x)=(x-3)(x-4)(x+1)(x+2)$$

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & -7 & 22 & 24 \\ & \downarrow & -1 & 5 & 2 & -24 \\ \hline -2 & 1 & -5 & -2 & 24 & 0 \\ & \downarrow & -2 & 14 & -24 & \\ \hline & 1x^2 & -7x & +12 & & 0 \end{array}$$

Find all the zeros $f(x)=x^3-2x^2-8x$

$$f(x)=x(x^2-2x-8)$$

$$0=x(x-4)(x+2)$$

$$\boxed{x=0,4,-2}$$

$x=$

Find all the zeros of: $2x^4-7x^3-8x^2+14x+8$

Find all the zeros of: $f(x)=x^3+x^2-14x+6$

Find the polynomial function with a leading coefficient of 2 that has the given degree and zeros: degree 3, zeros -2, 4, 1

$$\boxed{f(x) = 2(x+2)(x-4)(x-1)}$$