Secondary III 7-2 HW

Name: Key

Asymptotes of Rational Functions

State the domain using interval notation. For any *x-value* excluded from the domain, state whether the graph has a vertical asymptote or a "hole" at that *x-value*. Use a graphing calculator to check your answer.

1. $f(x) = \frac{x+5}{x+1}$

D: $(-\infty, -1) \cup (-1, \infty)$

VA: $x = -1$

2. $f(x) = \frac{x^2 + 2x - 3}{x^2 - 4x + 3}$

$\frac{(x+3)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}}$

D: $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$

hole: $x = 1$

VA: $x = 3$

Find any holes, asymptotes, and intercepts and state the end behavior.

3. $f(x) = \frac{x-1}{x^2+x-6}$

$(x+3)(x-2)$

VA: $x = 3, 2$

HA: $y = 0$

x-int: $(1, 0)$

y-int: $(0, 1/6)$

$\lim_{x \to \pm\infty} f(x) = 0$

7. $f(x) = \frac{x-1}{x+1}$

VA: $x = -1$

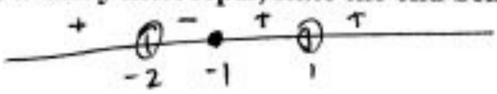
HA: $y = 1$

xint: $(1, 0)$

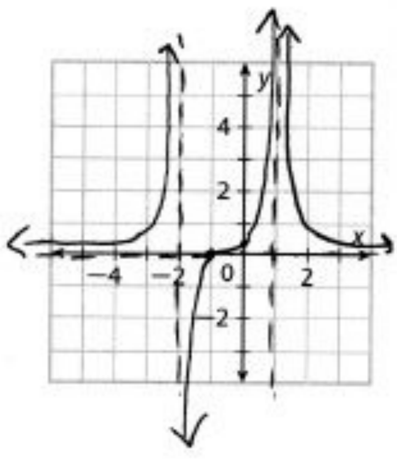
yint: $(0, -1)$

$\lim_{x \to \pm\infty} f(x) = 1$

Sketch the graph of the given rational function. Also state the function's domain and range using interval notation. Find any x and y intercepts, state the end behavior, and behavior around the asymptotes.



8. $f(x) = \frac{x+1}{(x-1)^2(x+2)}$



Domain: $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

Range: $(-\infty, \infty)$

X – intercept: $(-1, 0)$

Y – intercept: $(0, 1/2)$

V Asymptote: $x = 1, -2$

Hole: n/a

increasing: $(-\infty, -2) \cup (-2, 1)$

decreasing: $(1, \infty)$

End Behavior:

$\lim_{x \to \pm\infty} f(x) = 0$

Asymptotes Behavior:

$\lim_{x \to -2^+} f(x) = -\infty$

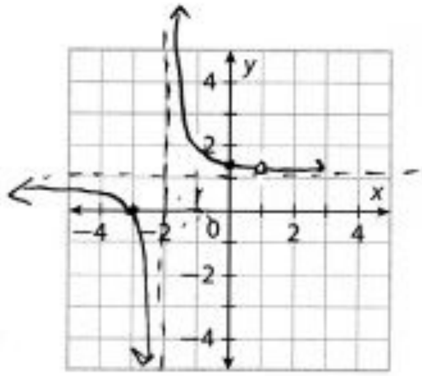
$\lim_{x \to -2^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = \infty$

$\lim_{x \to 1^-} f(x) = \infty$

9. $f(x)=\frac{x^2+2x-3}{x^2+x-2}$ $=\frac{(x+3)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}$

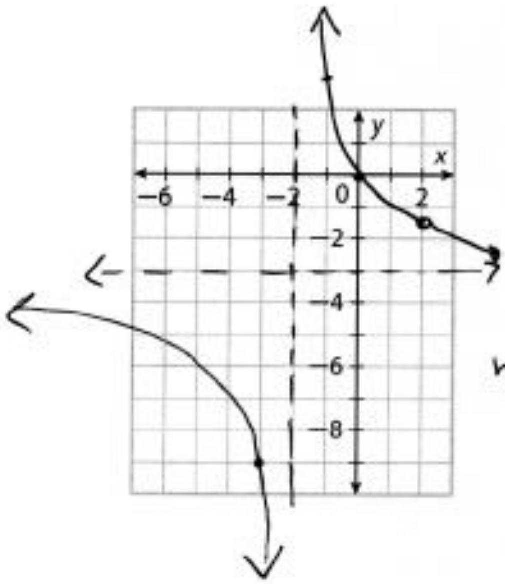
hole @ $(1, 4/3)$



Domain: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$
Range: $(-\infty,1)\cup(1,4/3)\cup(4/3,\infty)$
X – intercept: $(-3,0)$
Y – intercept: $(0,3/2)$
V Asymptote: $x=-2$
Hole: $x=1$
increasing: n/a
decreasing: $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$
End Behavior:
$\lim_{x\to\pm\infty} f(x)=1$

Asymptotes Behavior:
$\lim_{x\to -2^-} f(x)=-\infty$ $\lim_{x\to -2^+} f(x)=\infty$

10. $f(x)=\frac{-3x\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$



x	y
-1	3
-3	-9
~~2~~	-3/2

hole @ $(2,-\tfrac{3}{2})$

Domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$
Range: $(-\infty,-3)\cup(-3,-3/2)\cup(-3/2,\infty)$
X – intercept: $(0,0)$
Y – intercept: $(0,0)$
V Asymptote: $x=-2$
Hole: $x=2$
Increasing: n/a
Decreasing: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$
End Behavior:
$\lim_{x\to\pm\infty} f(x)=-3$

Asymptotes Behavior:
$\lim_{x\to -2^-} f(x)=-\infty$

$\lim_{x\to -2^+} f(x)=\infty$

$x-2+\frac{3}{x+2}$

$x+2 \overline{) x^2+0x-1}$

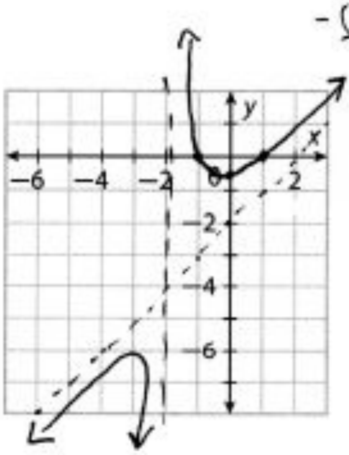
$-(x^2+2x) \downarrow$

$-2x-1$

$-(-2x-4)$

3

11. $f(x)=\frac{x^2-1}{x+2} = \frac{(x+1)(x-1)}{(x+2)}$



$-2 \,|\; 1 \quad 0 \quad -1$

$\downarrow \quad -2 \quad 4$

$1x - 2 + \frac{3}{x+2}$

use calc

Domain: $(-\infty,-2)\cup(-2,\infty)$

Range: $(-\infty,-7.5)\cup(-0.5,\infty)$

X – intercept: $(1,0)\ (-1,0)$

Y – intercept: $(0,-1/2)$

V Asymptote: $x=-2$

Hole: n/a

increasing: $(-\infty,-3.7)\cup(-0.3,\infty)$

decreasing: $(-3.7,-2)\cup(-2,-0.3)$

End Behavior:

$\lim_{x\to-\infty} f(x) = -\infty \quad \lim_{x\to\infty} f(x) = \infty$

Asymptotes Behavior:

$\lim_{x\to-2^-} f(x) = -\infty \quad \lim_{x\to-2^+} f(x) = \infty$

18. Draw Conclusions For what value(s) of a does the graph of $f(x) = \frac{x+a}{x^2+4x+3}$ $(x+3)(x+1)$ have a "hole"? Explain. Then, for each value of a, state the domain and the range of $f(x)$ using interval notation. $a = 3$ or 1

$\frac{x+3}{(x+3)(x+1)}$

3

D: $(-\infty,-3)\cup(-3,\infty)$

R: $(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},0)\cup(0,\infty)$

1

$\frac{x+1}{(x+3)(x+1)}$

D: $(-\infty,-1)\cup(-1,\infty)$

R: $(-\infty,0]\cup(0,\frac{1}{2})\cup(\frac{1}{2},\infty)$

19. Critique Reasoning A student claims that the functions $f(x) = \frac{4x^2-1}{4x+2}$ and $g(x) = \frac{4x+2}{4x^2-1}$ have different domains (correct) but identical ranges (false). Which part of the student's claim is correct, and which is false? Explain.

$\frac{(2x+1)(2x-1)}{2(2x+1)}$

D: $(-\infty,-1/2)\cup(-1/2,\infty)$

R: $(-\infty,0)\cup(0,\infty)$

$\frac{2(2x+1)}{(2x+1)(2x-1)}$ D: $(-\infty,\frac{-1}{2})\cup(\frac{-1}{2},\frac{1}{2})\cup(\frac{1}{2},\infty)$

R: $(-\infty,-1)\cup(-1,0)\cup(0,\infty]$

Review

Simplify the following rational expressions * change to division

1. $\frac{2}{x^2-x-2} \div \frac{10}{x^2+2x-8}$ $\boxed{x \neq 2,-1,-4}$

$\frac{1\ \cancel{2}}{(\cancel{x-2})(x+1)} \cdot \frac{(x+4)(\cancel{x-2})}{5\ \cancel{10}} = \boxed{\frac{x+4}{5(x+1)}}$

2. $\frac{x}{x^2-6x+8} \div \frac{1}{x^2-x-12}$ $\boxed{x \neq 2,\ 4,-3}$

$\frac{x}{(\cancel{x-4})(x-2)} \cdot \frac{(\cancel{x-4})(x+3)}{1} = \boxed{\frac{x(x+3)}{(x-2)}}$