7-2a: Asymptotes of Rational Functions

Objectives:

1. I can find the holes and vertical asymptotes of a rational function.
2. I can find the x- and y-intercepts of a rational function.
3. I can find the end behavior models of a rational function.
4. I can analyze a graph of a rational function.
5. I can graph a rational function by hand.

Holes and Vertical asymptotes - zeros of denomina for

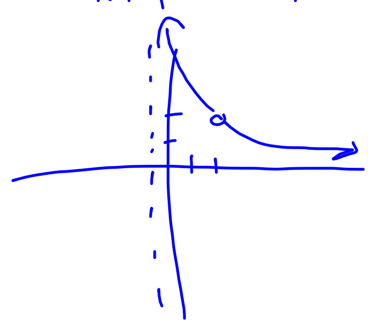
→ ones that cancel → everything else

$$f(x)=\frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}$$

$$\frac{x+3}{x+1}=\frac{2+3}{2+1}=\frac{5}{3}$$

hole: $x=2$ $(2, 5/3)$

VA: $x=-1$



Find the holes and vertical asymptotes

a. $y = \frac{5x}{x+2}$

holes: n/a

VA: x = -2

b. $y = \frac{2x^3}{x-5}$

holes: n/a

VA: x = 5

c. $y = \frac{\cancel{x+2}}{(x-2)\cancel{(x+2)}}$

hole: x = -2 $(-2, -\frac{1}{4})$

VA: x = 2

d. $f(x) = \frac{x^2-9}{x^2-5x+6}$

$\frac{(x+3)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}}$

hole: x = 3

VA: x = 2

X and Y Intercepts

X intercepts, y = 0

set numerater = 0 and solve for x.

$f(x) = \frac{3x-12}{x^2-5x-6}$

$3x - 12 = 0$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

$\boxed{(4, 0)}$

Y intercepts, $\boxed{x = 0}$

$f(x) = \frac{3x-12}{x^2-5x-6}$

$f(0) = \frac{3(0)-12}{0^2-5(0)-6} = \frac{-12}{-6} = 2$

$\boxed{(0, 2)}$

Find the x and y intercepts of the following functions:

$$f(x)=\frac{x^2-2x-3}{x+2}$$

$$f(x)=\frac{(x-3)(x+1)}{x+2}$$

x-int: (3,0), (-1,0)

y-int: (0, -3/2)

$$f(x)=\frac{3x-5}{x^2-5x+6}$$

$$f(x)=\frac{3x-5}{(x-2)(x-3)}$$

x-int: (5/3, 0)

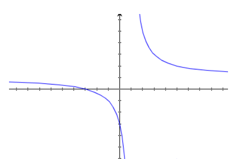
y-int: (0, -5/6)

Horizontal Asymptotes
(End Behavior Models)

Look at the graphs, see if you can find the end behavior models. What are the patterns?

$$f(x)=\frac{x+3}{x-1}$$

HA: y=1



$$f(x)=\frac{(x+5)(x-1)}{x+1}$$

$$\frac{x^2+4x-5}{x+1}$$

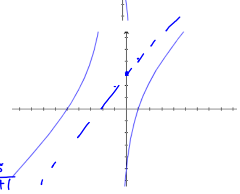
$$x+3+\frac{-8}{x+1}$$

$$x+1\,\overline{)\,x^2+4x-5}$$

$$-(x^2+x)$$

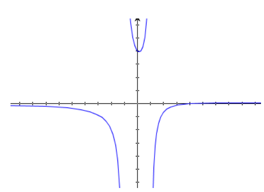
$$3x-5$$

$$3x+3$$



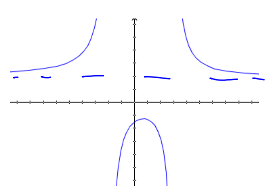
$$f(x)=\frac{x-4}{(x+1)(x-1)}$$

HA: y=0



$$f(x)=\frac{2x^2-3x+9}{x^2-x-6}$$

y=2



Horizontal Asymptotes (End Behavior):

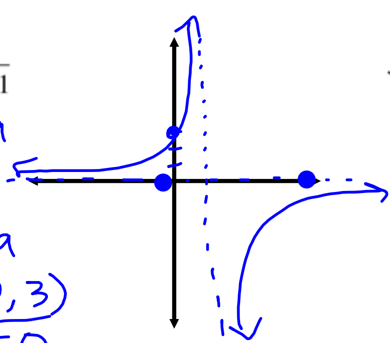
To find the Horizontal Asymptote (end behavior model), compare the degrees of the numerator and denominator.

Top heavy: y = slant asymptote - long division
Bottom heavy: y = 0
Equal: y = divide leading coefficients

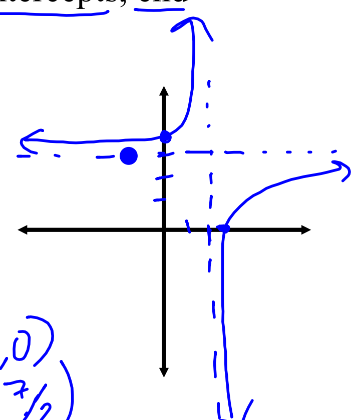
Identify the holes, vertical asymptotes, x and y intercepts, end behavior, and then make a sketch. HA

$f(x) = \frac{-3}{x-1}$

holes: n/a
VA: x=1
xint: n/a
yint: (0, 3)
HA: y=0



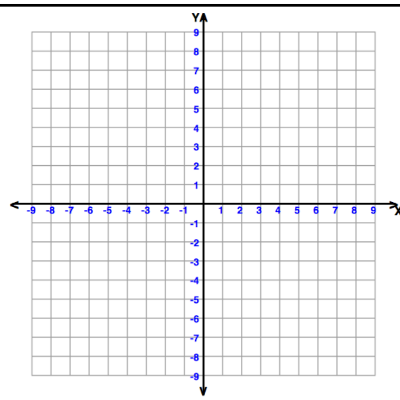
$f(x) = \frac{3x-7}{x-2}$



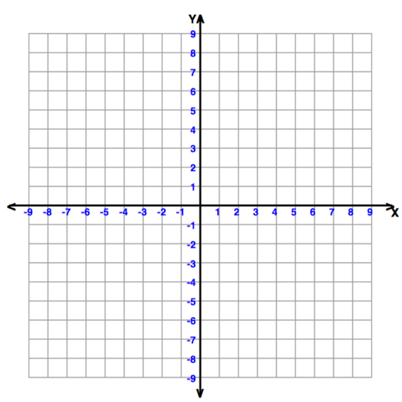
VA: x=2
xint: (7/3, 0)
yint: (0, 7/2)
HA: y=3

7-2b Graphing Rational Functions

$$f(x) = \frac{3x - 2}{x - 1}$$



$$f(x) = \frac{-(x+3)}{(x-1)^2(x+2)}$$



Non-Horizontal End Behavior

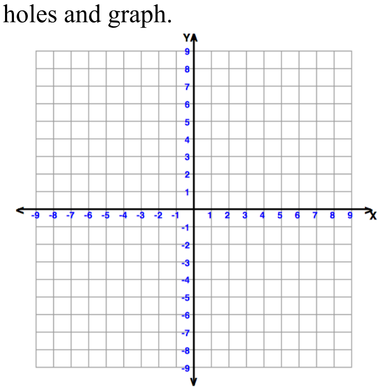
Top heavy rational functions have non-horizontal end behaviors

To find the degree of the end behavior **model (EBM)** - divide the leading terms and reduce.

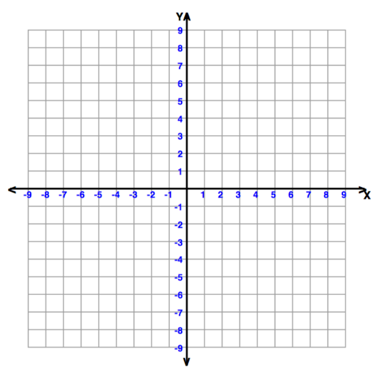
the ends of $\dfrac{3x^5-4x^2+5}{2x^3-5x+4}$ will behave like $\dfrac{3x^5}{2x^3}=\boxed{\dfrac{3x^2}{2}}$

Ex. 5 Find all asymptotes/EBM, holes and graph.

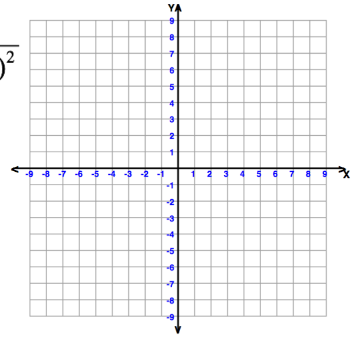
a. $f(x)=\dfrac{x^3}{x^2-9}$



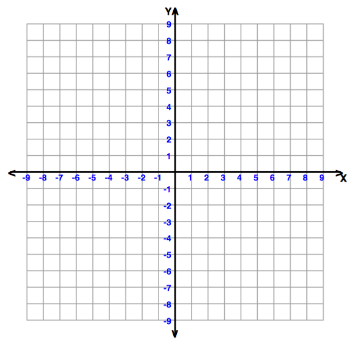
b. $f(x)=\dfrac{x^2-9}{x^2-5x+6}$



c. $f(x)=\dfrac{-(x+1)(x+2)}{(x+3)(x+2)(x-1)^2}$

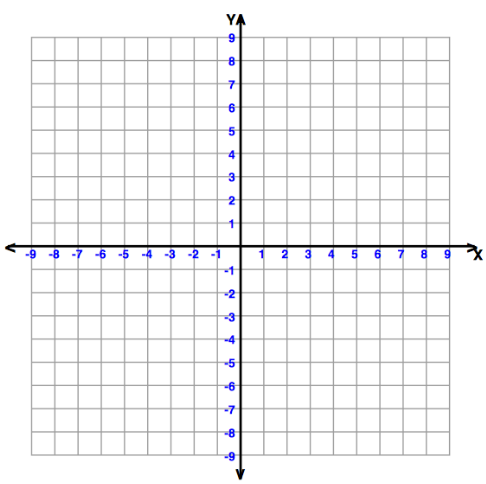


d. $f(x)=\dfrac{x(x+3)(x+2)}{x^2-4}$



Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

$$f(x)=\frac{x-1}{x^2-x-12}$$



Domain
Range
x-intercepts
y-intercepts
VA
HA
Increasing
Decreasing
Continuous
Asymptote Behavior

End Behavior