7-2a: Asymptotes of Rational Functions

Objectives:

1. I can find the holes and vertical asymptotes of a rational function.
2. I can find the x- and y-intercepts of a rational function.
3. I can find the end behavior models of a rational function.
4. I can analyze a graph of a rational function.
5. I can graph a rational function by hand.

Holes and Vertical asymptotes

$$f(x)=\frac{(x+3)(x-2)}{(x-2)(x+1)}$$

Find the holes and vertical asymptotes

a. $y = \dfrac{5x}{x+2}$

b. $y = \dfrac{2x^3}{x-5}$

c. $y = \dfrac{x+2}{(x-2)(x+2)}$

d. $f(x) = \dfrac{x^2-9}{x^2-5x+6}$

X and Y Intercepts

Y intercepts, x = 0

$$f(x) = \frac{3x-12}{x^2-5x-6}$$

X intercepts, y = 0

$$f(x) = \frac{3x-12}{x^2-5x-6}$$

Find the x and y intercepts of the following functions:

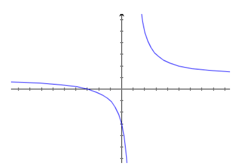
$$f(x)=\frac{x^2-2x-3}{x+2}$$

$$f(x)=\frac{3x-5}{x^2-5x+6}$$

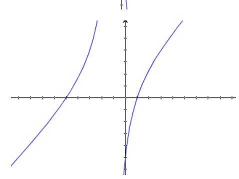
Horizontal Asymptotes
(End Behavior Models)

Look at the graphs, see if you can find the end behavior models. What are the patterns?

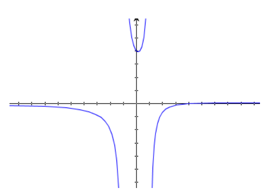
$$f(x)=\frac{x+3}{x-1}$$



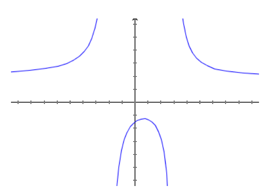
$$f(x)=\frac{(x+5)(x-1)}{x+1}$$



$$f(x)=\frac{x-4}{(x+1)(x-1)}$$



$$f(x)=\frac{2x^2-3x+9}{x^2-x-6}$$



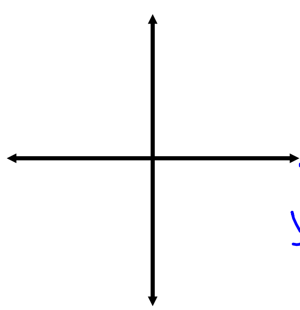
Horizontal Asymptotes (End Behavior):

To find the Horizontal Asymptote (end behavior model), compare the degrees of the numerator and denominator.

- **Top heavy:** y = slant asymptote
- **Bottom heavy:** y = 0
- **Equal:** y = divide leading coefficients

Identify the holes, vertical asymptotes, x and y intercepts, end behavior, and then make a sketch.

$f(x) = \frac{-3}{x-1}$



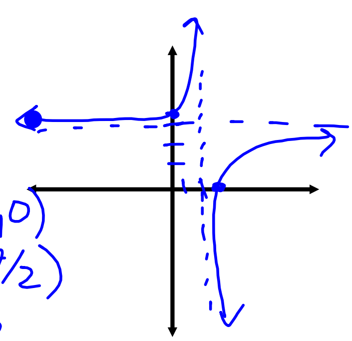
$f(x) = \frac{3x-7}{1x-2}$

VA: $x = 2$

x-int: $(7/3, 0)$

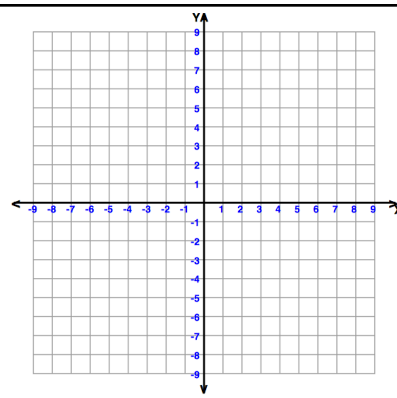
y int: $(0, 7/2)$

HA: $y = 3$



7-2b Graphing Rational Functions

$$f(x)=\frac{3x-2}{x-1}$$



$$f(x)=\frac{-(x+3)}{(x-1)^2(x+2)}$$

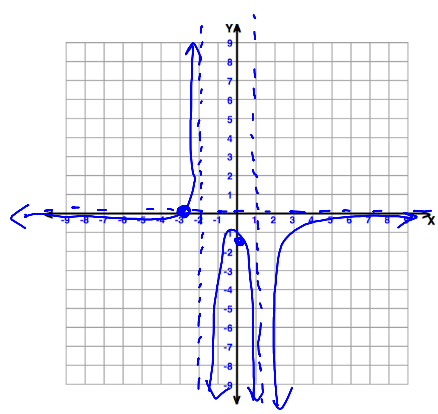
VA: $x=1,\ x=-2$

EBM/HA: $y=0$

x-int: $(-3,0)$

y-int: $(0, \frac{-3}{2})$

$f(0)=$



Non-Horizontal End Behavior

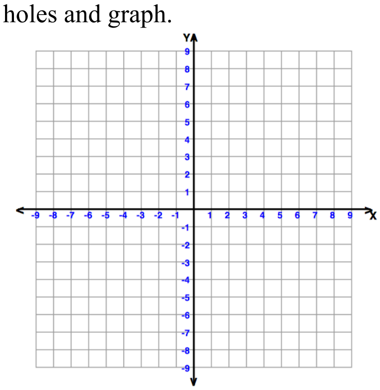
Top heavy rational functions have non-horizontal end behaviors

To find the degree of the end behavior **model (EBM)** - divide the leading terms and reduce.

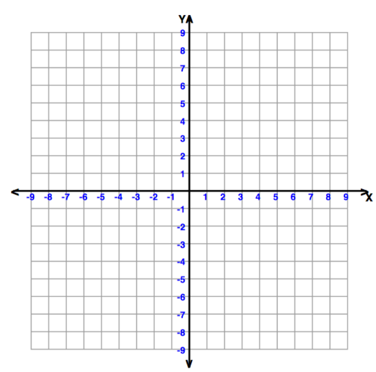
the ends of $\frac{3x^5 - 4x^2 + 5}{2x^3 - 5x + 4}$ will behave like $\frac{3x^5}{2x^3} = \boxed{\frac{3x^2}{2}}$

Ex. 5 Find all asymptotes/EBM, holes and graph.

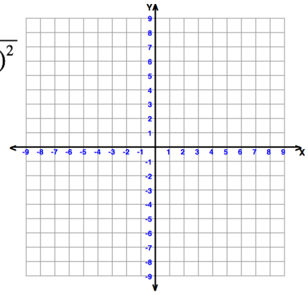
a. $f(x) = \frac{x^3}{x^2 - 9}$



b. $f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$



c. $f(x)=\dfrac{-(x+1)(x+2)}{(x+3)(x+2)(x-1)^2}$



Top: long division

d. $f(x)=\dfrac{x(x+3)(x+2)}{x^2-4}$

$(x+2)(x-2)$

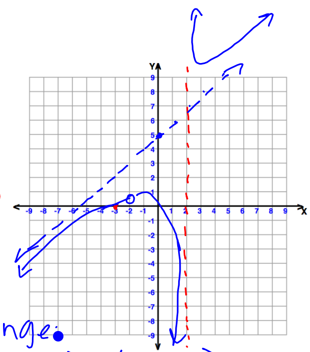
hole: $x=-2$ $(-2, \frac{1}{2})$

VA: $x=2$

x-int: $(0,0)(-3,0)$

y-int: $(0,0)$

EBM: $y=x+5$



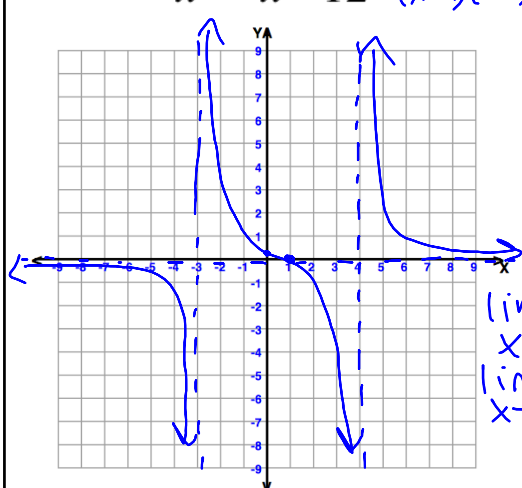
Range: $(-\infty,1)\cup(10,\infty)$

Inc: $(-\infty,-2)\cup(-2,-1)\cup(3,\infty)$

Dec: $(-1,2)\cup(2,3)$

Find the intercepts, asymptotes, limits at vertical asymptotes, analyze and draw the graph of

$$f(x)=\frac{x-1}{x^2-x-12}=\frac{x-1}{(x-4)(x+3)}$$



Domain $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

Range $(-\infty,\infty)$

x-intercepts $(1,0)$

y-intercepts $(0,\frac{1}{12})$

VA : $x=4$, $x=-3$

HA: $y=0$

Increasing n/a

Decreasing $(-\infty,-3)\cup(-3,4)\cup(4,\infty)$

Continuous yes

Asymptote Behavior

$\lim_{x\to -3^-} f(x) = -\infty$ $\lim_{x\to 4^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$ $\lim_{x\to 4^+} f(x) = \infty$

End Behavior

$\lim_{x\to \pm\infty} f(x) = 0$