

Solving Rational Inequalities

1. $\frac{5(x-1) + 3(x+3)}{(x+3)(x-1)} < 0$

points of interest:
 $x\text{-int: } -\frac{1}{2}$
 $VA: x = -3, 1$

$\frac{5x - 5 + 3x + 9}{(x+3)(x-1)} < 0$

$\frac{8x + 4}{(x+3)(x-1)} < 0$

$\frac{4(x+1)}{(x+3)(x-1)} < 0$

Sign chart: $\leftarrow \oplus \oplus \ominus \oplus \rightarrow$ (at $x = -3, -\frac{1}{2}, 1$)

Sign test:
 $-4: \frac{-}{-} = \frac{-}{+} = -$
 $-1: \frac{-}{+} = -$
 $0: \frac{+}{+} = +$
 $2: \frac{+}{+} = +$

Solution: $(-\infty, -3) \cup (-\frac{1}{2}, 1)$

3. $\frac{x+2}{x^2-9} < 0$

$\frac{x+2}{(x+3)(x-3)} < 0$

Sign chart: $\leftarrow \oplus \oplus \ominus \oplus \rightarrow$ (at $x = -3, -2, 3$)

Sign test:
 $-4: \frac{-}{+} = -$
 $-2.5: \frac{-}{-} = +$
 $0: \frac{+}{-} = -$
 $4: \frac{+}{+} = +$

Solution: $(-\infty, -3) \cup (-2, 3)$

5. $\frac{x^2+x-12}{x^2-4x+4} > 0$

$\frac{(x+4)(x-3)}{(x-2)^2} > 0$

Sign chart: $\leftarrow \oplus \oplus \oplus \oplus \rightarrow$ (at $x = -4, 2, 3$)

Sign test:
 $-5: \frac{-}{+} = -$
 $0: \frac{+}{+} = +$
 $2.5: \frac{+}{+} = +$
 $4: \frac{+}{+} = +$

Solution: $(-\infty, -4) \cup (3, \infty)$

7. $\frac{1}{x+1} \leq \frac{1}{x-3}$

$\frac{1(x-3) - 1(x+1)}{(x+1)(x-3)} \leq 0$

$\frac{x-3-x-1}{(x+1)(x-3)} \leq 0 \rightarrow \frac{-2x-4}{(x+1)(x-3)} \leq 0$

$\frac{-2(x+2)}{(x+1)(x-3)} \leq 0$

Sign chart: $\leftarrow \oplus \oplus \oplus \oplus \rightarrow$ (at $x = -1, -2, 3$)

Sign test:
 $2: \frac{-}{-} = +$
 $4: \frac{+}{+} = +$
 $0: \frac{-}{-} = +$

Solution: $(-\infty, -1) \cup [1, 3)$

2. $\frac{x-1}{x^2-4} < 0$

$\frac{x-1}{(x+2)(x-2)} < 0$

Sign chart: $\leftarrow \oplus \oplus \oplus \oplus \rightarrow$ (at $x = -2, 1, 2$)

Sign test:
 $-3: \frac{-}{-} = +$
 $0: \frac{+}{-} = -$
 $1.5: \frac{+}{-} = -$
 $3: \frac{+}{+} = +$

Solution: $(-\infty, -2) \cup (1, 2)$

4. $\frac{x^2-1}{x^2+1} \leq 0$

$\frac{(x+1)(x-1)}{x^2+1} \leq 0$

Sign chart: $\leftarrow \oplus \oplus \oplus \oplus \rightarrow$ (at $x = -1, 1$)

Sign test:
 $-2: \frac{-}{+} = -$
 $0: \frac{+}{+} = +$
 $2: \frac{+}{+} = +$

Solution: $[-1, 1]$

6. $\frac{x^3-4x}{x^2+2} \leq 0$

$\frac{x(x+2)(x-2)}{x^2+2} \leq 0$

Sign chart: $\leftarrow \oplus \oplus \oplus \oplus \rightarrow$ (at $x = -2, 0, 2$)

Sign test:
 $-3: \frac{-}{-} = +$
 $-1: \frac{-}{+} = -$
 $1: \frac{+}{+} = +$
 $3: \frac{+}{+} = +$

Solution: $(-\infty, -2] \cup [0, 2)$

8. $\frac{1}{x+2} > \frac{2}{x-1}$

$\frac{1(x-1) - 2(x+2)}{(x+2)(x-1)} > 0$

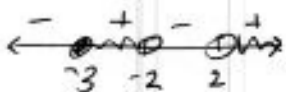
$\frac{x-1-2x-4}{(x+2)(x-1)} > 0 \rightarrow \frac{-x-5}{(x+2)(x-1)} > 0$

Sign chart: $\leftarrow \oplus \oplus \oplus \oplus \rightarrow$ (at $x = -5, -2, 1$)

Sign test:
 $-6: \frac{+}{-} = -$
 $0: \frac{-}{-} = +$

Solution: $(-\infty, -5) \cup (-2, 1)$

$$9. \frac{x+3}{x^2-4} \geq 0 \quad \frac{x+3}{(x+2)(x-2)} \geq 0$$



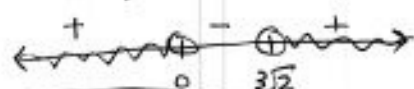
- 4: $\frac{-}{-} = +$
 2: $\frac{+}{-} = -$
 0: $\frac{+}{-} = -$
 3: $\frac{+}{+} = +$

$$[-3, -2) \cup (2, \infty)$$

$$* 11. \frac{2}{x} + \frac{x^2}{1} > 0 \quad -2 + x^3 = 0$$

$$\sum x^3 = \sqrt[3]{2} \quad x = \sqrt[3]{2} = 1.26$$

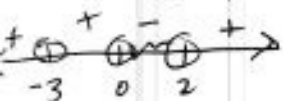
$$\frac{-2 + x^3}{x} > 0$$



- 1: $\frac{-}{-} = +$
 1: $\frac{-}{+} = -$
 2: $\frac{+}{+} = +$

$$(-\infty, 0) \cup (\sqrt[3]{2}, \infty)$$

$$13. \frac{x^3(x-2)}{(x+3)^2} < 0$$



- 4: $\frac{-}{+} = -$
 -1: $\frac{-}{-} = +$
 1: $\frac{+}{+} = +$
 3: $\frac{+}{+} = +$

$$(0, 2)$$

Review

$$1. \text{ Solve the equation } \frac{3 \cdot 5x(x+2)}{x+2} - \frac{1}{x} = \frac{1}{5x}$$

$$15x - 5x - 10 = x + 2$$

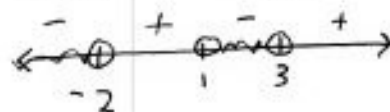
$$10x - 10 = x + 2$$

$$9x = 12$$

$$x = \frac{12}{9}$$

$$x = \frac{4}{3}$$

$$10. \frac{x^2-7}{x^2-x-6} < 1 \quad \frac{x^2-7}{(x-3)(x+2)} - \frac{1}{(x^2-x-6)} < 0$$

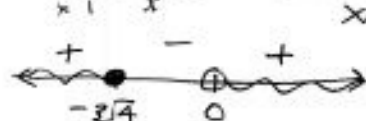


$$\frac{x-1}{(x-3)(x+2)} < 0$$

- 3: $\frac{-}{-} = +$
 0: $\frac{-}{+} = -$
 2: $\frac{+}{-} = -$
 1: $\frac{+}{+} = +$

$$(-\infty, -2) \cup (1, 3)$$

$$* 12. \frac{x \cdot x^2 + \frac{4}{x}}{x} \geq 0 \quad \frac{x^3 + 4}{x} \geq 0$$

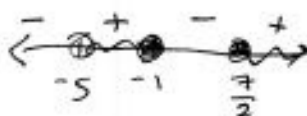


$$\sqrt[3]{x^3} = \sqrt[3]{-4} \quad x = -\sqrt[3]{4} = -1.6$$

- 2: $\frac{-}{-} = +$
 -1: $\frac{-}{+} = -$
 1: $\frac{+}{+} = +$

$$(-\infty, -\sqrt[3]{4}] \cup (0, \infty)$$

$$14. \frac{(2x-7)(x+1)}{x+5} \geq 0$$



- 6: $\frac{-}{-} = +$
 -2: $\frac{-}{+} = -$
 0: $\frac{-}{+} = -$
 4: $\frac{+}{+} = +$

$$(-5, -1] \cup [7/2, \infty)$$

$$2. \text{ Find the inverse of } f(x) = \frac{x-2}{x+2}$$

$$x = \frac{y-2}{y+2}$$

$$xy + 2x = y - 2$$

$$xy - y = -2 - 2x$$

$$y(x-1) = -2 - 2x$$

$$y = \frac{-2 - 2x}{x-1} \quad \text{or}$$

$$2x+2 = y - xy$$

$$2x+2 = y(1-x)$$

$$\frac{2x+2}{1-x} = y$$