Secondary 3 Math

Name Key

8-3 Modeling with Exponential Functions

Part 1: Determine the exponential function that models the situation.

1. Initial value = 5, increasing at a rate of 17% per year.

$$f(t) = 5(1+.17)^t \rightarrow 5(1.17)^t$$

2. Initial value = \$4000, decreasing at a rate of 5.5% per year.

$$f(t) = 4000(1-.055)^t \rightarrow 4000(.945)^t$$

3. Initial mass = 0.6 g. , doubling every hour

$$f(t) = .6(2)^t$$

4. Initial mass = 8 g. , halving once every 5700 years.

$$f(t) = 8\left(\tfrac{1}{2}\right)^{t/5700}$$

5. You buy a video game console for \$500 and sell it 5 years later for \$100. The resale value decays exponentially over time. Write a function that represents the resale value, R, in dollars over the time, t, in years.

$100 = 500(1-r)^5$

$.2 = (1-r)^5 \quad r = .275$

$$R(t) = 500(.725)^t$$

Part 2: Growth and Decay problems:

6. The 2000 population of Lehi was 26,000, and was increasing at a rate of 8.5% per year. Predict the population of Lehi in 2015. When will the population reach 100,000?

$P(t) = 26{,}000(1+.085)^t$

$t=15 \Rightarrow \boxed{88{,}393 \text{ people}}$

$100{,}000 = 2600(1.085)^t$

$\boxed{t = 16.5 \text{ years}}$

7. The half-life of strontium -90 is 28.8 years. How long will it take a 10 gram sample to decay to less than 1 gram?

$1 = 10\left(\tfrac{1}{2}\right)^{t/28.8}$

$t = \boxed{95.7 \text{ years}}$

8. The George River herd of caribou in Canada was estimated to be about 4700 in 1954 and grew at an exponential rate to about 472,000 in 1984. Use the exponential growth function of $P(t) = P_0 e^{0.154t}$, where P_0, is the initial population, t is the time in years after 1954, and $P(t)$ is the population at time t, to determine how many years after 1984 the herd reached 25 million.

$25\,000\,000 = 4700e^{0.154t}$

$t = 55.71$ years $\boxed{56 \text{ years} \ldots\ 2010}$

Part 3: Money

7. If Hugh invests \$1500 at 4% compounded annually, how much money will he have after 7 years?

$$1500\left(1+\frac{.04}{1}\right)^{7(1)} = \boxed{\$1973.90}$$

8. If Bob invests \$2400 at 3.6% compounded annually, how long will it take him to double his money?

$$4800 = 2400\left(1+\frac{.036}{1}\right)^{t(1)}$$

$$t = \boxed{19.6 \text{ years later}}$$

$A = Pe^{rt}$

9. How much money will you have after 6 years if you invest \$1000 at 5% interest compounded continuously?

$1000e^{.05(6)} = \boxed{\$1349.86}$

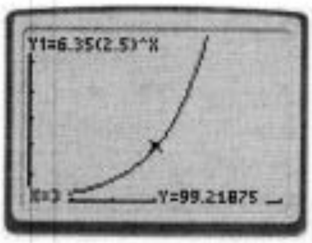
10. What interest rate do you need to double an investment in 7 years if it is compounded continuously?

$2 = 1e^{r \cdot 7}$ $r = .099$ $r = 1\%$

11. Which investment is more attractive, 5% compounded monthly or 5.1% compounded quarterly?

$1\left(1 + \frac{.051}{4}\right)^{4(1)} = 1.05198$ $1\left(1 + \frac{.05}{12}\right)^{12 \cdot 1} = 1.05116$

24. Explain the Error A student has a baseball card that is worth \$6.35. He looks up the appreciation rate and finds it to be 2.5% per year. He wants to find how much it will be worth after 3 years. He writes the function $f(t) = 6.35(2.5)^t$ and uses the graph of that function to find the value of the card in 3 years.



According to his graph, his card will be worth about \$99.22 in 3 years. What did the student do wrong? What is the correct answer?

$f(t) = 6.35(1.025)^t$

$= \boxed{\$6.84}$

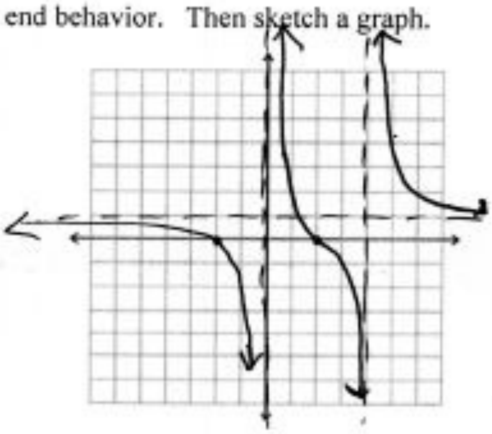
Review

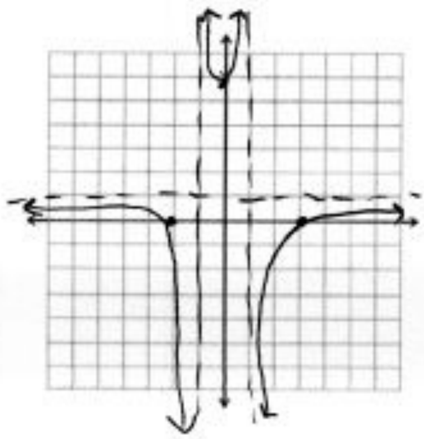
Find any holes, asymptotes, and intercepts and state the end behavior. Then sketch a graph.

$\frac{21}{5}$

1. $f(x) = \dfrac{x^2 - 4}{x^2 - 4x} = \dfrac{(x+2)(x-2)}{x(x-4)}$

holes: none
VA: $x = 0, 4$
xint: $(-2, 0)$ $(2, 0)$
yint: none
$y = 1$





2. $g(x) = \dfrac{x^2 - x - 6}{x^2 - 1} = \dfrac{(x-3)(x+2)}{(x+1)(x-1)}$

VA: $x = \pm 1$
HA: $y = 1$
xint: $(3, 0)$ $(-2, 0)$
yint: $(0, 6)$