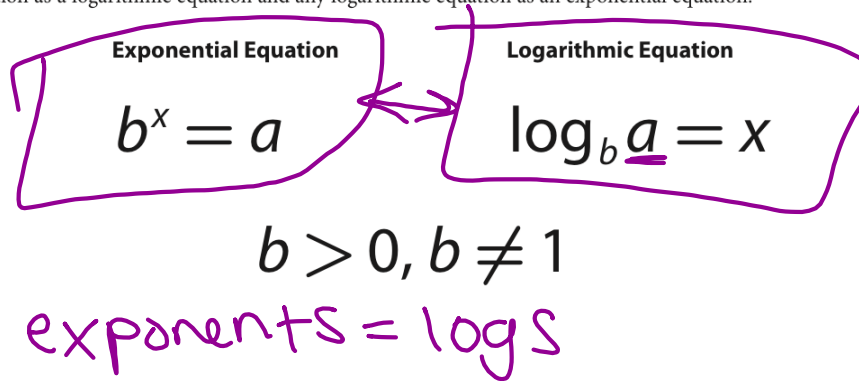


9-1 Defining and evaluating logarithms

Explain 1 Converting Between Exponential and Logarithmic Forms of Equations

In general, the exponential function $f(x) = b^x$, where $b > 0$ and $b \neq 1$, has the logarithmic function $f^{-1}(x) = \log_b x$ as its inverse. For instance, if $f(x) = 3^x$, then $f^{-1}(x) = \log_3 x$, and if $f(x) = \left(\frac{1}{4}\right)^x$, then $f^{-1}(x) = \log_{\frac{1}{4}} x$. The inverse relationship between exponential functions and logarithmic functions also means that you can write any exponential equation as a logarithmic equation and any logarithmic equation as an exponential equation.



Examples

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Exponential Equation	Logarithmic Equation
$4^3 = 64$	$\log_4 64 = 3$
$5^{-2} = \frac{1}{25}$	$\log_5 \frac{1}{25} = -2$
$\left(\frac{2}{3}\right)^p = q$	$\log_{\frac{2}{3}} q = p$
$\left(\frac{1}{2}\right)^n = m$	$\log_{\frac{1}{2}} m = n$

Switch between Log and exponential forms

Exponential Equation	Logarithmic Equation
$3^5 = 243$	$5 = \log_3 243$
$4^3 = \frac{1}{64}$	$\log_4 \frac{1}{64} = -3$
$\left(\frac{3}{4}\right)^r = s$	$r = \log_{3/4} s$
$\left(\frac{1}{5}\right)^w = v$	$\log_{1/5} v = w$

The natural logarithm:

$$y = \ln x \text{ is equivalent to } x = e^y$$

$$y = \log_e x$$

The common logarithm:

$$y = \log x \text{ is equivalent to } x = 10^y$$

$$y = \log_{10} x$$

(not in the book)

Exponential Equation	Logarithmic Equation
$e^5 \approx 148.4$	$S = \ln 148.4$
$e^{1.8} = 6$	$\ln 6 \approx 1.8$
$10^5 = 100,000$	$S = \log 100000$
$10^3 = 1000$	$\log 1,000 = 3$

If $f(x) = \log_{10} x$, find $f(1000)$, $f(0.01)$, and $f(\sqrt{10})$.

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$$f(1000) = x$$

$$f(0.01) = x \quad \text{P 795}$$

$$f(\sqrt{10}) = x$$

$$\log_{10} 1000 = x$$

$$\log_{10} 0.01 = x$$

$$\log_{10} \sqrt{10} = x$$

$$10^x = 1000$$

$$10^x = 0.01$$

$$10^x = \sqrt{10}$$

$$10^x = 10^3$$

$$10^x = 10^{-2}$$

$$10^x = 10^{\frac{1}{2}}$$

$$x = 3$$

$$x = -2$$

$$x = \frac{1}{2}$$

$$\text{So, } f(1000) = 3.$$

$$\text{So, } f(0.01) = -2.$$

$$\text{So, } f(\sqrt{10}) = \frac{1}{2}.$$

If $f(x) = \log_{\frac{1}{2}} x$, find $f(4)$, $f\left(\frac{1}{32}\right)$ and $f(2\sqrt{2})$.

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$$f(4) = x$$

$$\log_{\frac{1}{2}} 4 = x$$

$$\frac{1}{2}^x = 4$$

$$\frac{1}{2}^x = \left(\frac{1}{2}\right)^{-2}$$

$$\text{So, } f(4) = \boxed{-2}$$

$$f\left(\frac{1}{32}\right) = x$$

$$\log_{\frac{1}{2}} \frac{1}{32} = x$$

$$\frac{1}{2}^x = \frac{1}{32}$$

$$\frac{1}{2}^x = \left(\frac{1}{2}\right)^5$$

$$\text{So, } f\left(\frac{1}{32}\right) = \boxed{5}$$

$$f(2\sqrt{2}) = x$$

$$\log_{\frac{1}{2}} 2\sqrt{2} = x$$

$$\frac{1}{2}^x = 2\sqrt{2}$$

$$= 2^1 \cdot 2^{\frac{1}{2}}$$

$$= 2^{\frac{3}{2}}$$

$$\frac{1}{2}^x = 2^{\frac{3}{2}}$$

$$\frac{1}{2}^x = \left(\frac{1}{2}\right)^{-\frac{3}{2}}$$

$$\text{So } f(2\sqrt{2}) = \boxed{-\frac{3}{2}}$$

Your Turn

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9. If $f(x) = \log_7 x$, find $f(343)$, $f\left(\frac{1}{49}\right)$, and $f(\sqrt{7})$.

Find the exact value without a calculator

(not in the book)

$$\log_2 32 = x$$

$$2^x = 32 \quad x = \boxed{5}$$

$$2^x = 2^5$$

$$\log 10,000,000 = x$$

$$10^x = 10,000,000$$

$$10^x = 10^7$$

$$x = 7$$

$$\log_4 \frac{1}{16} = x$$

$$4^x = \frac{1}{16} \quad 4^x = (4)^{-2}$$

$$x = \boxed{-2}$$

$$\log .00001 = x$$

$$10^x = .00001$$

$$10^x = 10^{-5}$$

$$x = \boxed{-5}$$

You try

$$\log_5 25$$

$$\log_2 \frac{1}{8}$$

$$\log 1000$$

$$\log .001$$

Use a calculator to

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First, find the common logarithm of 0.42. Round the result to the thousandths place and raise 10 to that number to confirm that the power is close to 0.42.

$$\log 0.42 \approx \boxed{}$$

$$10^{\boxed{}} \approx 0.42$$

Next, find the natural logarithm of 0.42. Round the result to the thousandths place and raise e to that number to confirm that the power is close to 0.42.

$$\ln 0.42 \approx \boxed{}$$

$$e^{\boxed{}} \approx 0.42$$

Your Turn

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Use a scientific calculator to find the common logarithm and the natural logarithm of the given number. Verify each result by evaluating the appropriate exponential expression.

11. 0.25**12.** 4

$$\log 0.25 = -0.602$$

$$10^{-0.602} \approx 0.25 \checkmark$$

$$\ln 0.25 = -1.386$$

$$e^{-1.386} = 0.25$$

The acidity level, or pH, of a liquid is given by the formula $\text{pH} = \log \frac{1}{[\text{H}^+]}$ where $[\text{H}^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. In a typical chlorinated swimming pool, the concentration of hydrogen ions ranges from 1.58×10^{-8} moles per liter to 6.31×10^{-8} moles per liter. What is the range of the pH for a typical swimming pool?

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$$\text{pH} = \log \frac{1}{1.58 \times 10^{-8}} = 7.8$$

$$\text{pH} = \log \frac{1}{6.31 \times 10^{-8}} = 7.2$$

The intensity level L (in decibels, dB) of a sound is given by the formula $L = 10 \log \frac{I}{I_0}$ where I is the intensity (in watts per square meter, W/m^2) of the sound and I_0 is the intensity of the softest audible sound, about $10^{-12} \text{ W}/\text{m}^2$. What is the intensity level of a rock concert if the sound has an intensity of $3.2 \text{ W}/\text{m}^2$?

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