

Find the exact value of the following without a calculator

1. $\log_2 8 = \log_2 2^3 = \boxed{3}$

2. $\log_4 \left(\frac{1}{16}\right) = \log_4 4^{-2} = \boxed{-2}$

3. $\log 10,000,000,000,000 = \log 10^{13} = \boxed{13}$

4. $\log 0.000000001 = \log 10^{-9} = \boxed{-9}$

Write the given exponential equation in logarithmic form.

5. $5^3 = 125$
 $3 = \log_5 125$

6. $\left(\frac{1}{10}\right)^{-2} = 100$
 $-2 = \log_{\frac{1}{10}} 100$

7. $3^m = n$
 $m = \log_3 n$

8. $\left(\frac{1}{2}\right)^p = q$
 $p = \log_{\frac{1}{2}} q$

Write the given logarithmic equation in exponential form.

9. $\log_6 1296 = 4$
 $(6)^4 = 1296$

10. $\log_{\frac{1}{4}} \frac{1}{64} = 3$
 $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

11. $\log_8 x = y$
 $(8)^y = x$

12. $\log_{\frac{2}{3}} c = d$
 $\left(\frac{2}{3}\right)^d = c$

Write the given exponential equation in logarithmic form.

13. $10^5 = 100000$
 $5 = \log 100000$

14. $e^4 = 54.6$
 $4 = \ln 54.6$

Write the given logarithm in exponential form.

15. $\ln 5 \approx 1.6$
 $e^{1.6} = 5$

16. $\log 1,000,000 = 6$
 $10^6 = 1,000,000$

13. If $f(x) = \log_3 x$, find $f(243)$, $f\left(\frac{1}{27}\right)$, and $f(\sqrt{27})$.

$$\begin{aligned} f(243) &= \log_3 243 \\ &= \log_3 3^5 \\ &= \boxed{5} \end{aligned}$$

$$\begin{aligned} f\left(\frac{1}{27}\right) &= \log_3 \frac{1}{27} \\ &= \log_3 3^{-3} \\ &= \boxed{-3} \end{aligned}$$

$$\begin{aligned} f(\sqrt{27}) &= \log_3 \sqrt{27} \\ &= \log_3 (27)^{1/2} \\ &= \log_3 (3^3)^{1/2} \\ &= \log_3 3^{3/2} \\ &= \boxed{3/2} \end{aligned}$$

14. If $f(x) = \log_6 x$, find $f(36)$, $f\left(\frac{1}{6}\right)$ and $f(6\sqrt[3]{6})$

$$\begin{aligned} f(36) &= \log_6 36 \\ &= \log_6 6^2 \\ &= \boxed{2} \end{aligned}$$

$$\begin{aligned} f\left(\frac{1}{6}\right) &= \log_6 \left(\frac{1}{6}\right) \\ &= \log_6 6^{-1} \\ &= \boxed{-1} \end{aligned}$$

$$\begin{aligned} f(6\sqrt[3]{6}) &= \log_6 6\sqrt[3]{6} \\ &= \log_6 6^1 \cdot 6^{1/3} \\ &= \log_6 6^{4/3} \\ &= \boxed{4/3} \end{aligned}$$

Use a scientific calculator to find the common logarithm and the natural logarithm of the given number. Verify each result by evaluating the appropriate exponential expression.

16. 19

$$\begin{aligned} \log 19 &= 1.279 \\ \ln 19 &= 2.944 \end{aligned}$$

$$\begin{aligned} 10^{1.279} &= 19.0 \checkmark \\ e^{2.944} &= 19.0 \checkmark \end{aligned}$$

17. 9

$$\begin{aligned} \log 9 &= .954 & 10^{.954} &= 9 \checkmark \\ \ln 9 &= 2.197 & e^{2.197} &= 9 \checkmark \end{aligned}$$

18. 0.6

$$\begin{aligned} \log 0.6 &= -.222 \\ \ln 0.6 &= -.511 \end{aligned}$$

$$\begin{aligned} 10^{-.222} &= 0.6 \checkmark \\ e^{-.511} &= 0.6 \checkmark \end{aligned}$$

19. 0.31

$$\begin{aligned} \log 0.31 &= -.509 & 10^{-.509} &= 0.31 \checkmark \\ \ln 0.31 &= -1.171 & e^{-1.171} &= 0.31 \checkmark \end{aligned}$$

20. The acidity level, or pH, of a liquid is given by the formula $\text{pH} = \log \frac{1}{[\text{H}^+]}$ where $[\text{H}^+]$ is the concentration (in moles per liter) of hydrogen ions in the liquid. What is the pH of iced tea with a hydrogen ion concentration of 0.000158 mole per liter?

$$\text{pH} = \log \left(\frac{1}{0.000158} \right) = \boxed{3.8}$$



25. Justify Reasoning Evaluate each expression without using a calculator. Explain your reasoning.

a. $\ln e^2 = \boxed{2}$

b. $10^{\log 7} = \boxed{7}$

26. The loudness L , measured in decibels, of a sound of intensity x , measured in watts per square meter is $L(x) = 10 \log \frac{x}{10^{-12}}$. A whisper has an intensity level of 10^{-10} watt per square meter. How many decibels is a whisper?

$$L(10^{-10}) = 10 \log \left(\frac{10^{-10}}{10^{-12}} \right) = \boxed{20 \text{ decibels}}$$

Review

For the following sequences: a) Write a recursive rule, b) write an explicit rule, c) find the 10th term

1. $-4, 1, 6, 11, \dots$

2. $2, 6, 18, 54, \dots$

a) Recursive:

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$f(1) = 2$