9-2 Properties of Logarithims

$$\log_a 1 = 0 \qquad \log_a a^1 = 1$$

$a^0 = 1$ $\quad a^1 = a^1$

Evaluate

$\log_5 1 = 0$ $\qquad \ln 1 = 0$

$\log_4 4 = 1$ $\qquad \log 10 = 1$

Inverse Property of Logarithms
If b and M are positive real numbers, with $b \neq 0$, then

$$b^{\log_b M} = M$$

Evaluate

$12^{\log_{12}\sqrt{2}}$ ~~$= x$~~ $\qquad 10^{\log 0.2} = 0.2$

$\log_{12}\sqrt{2} = \log_{12} x$

Inverse Property of Logarithms
If b and r are positive real numbers, with $b \neq 0$, then

$$\log_a a^r = r$$

Evaluate

~~$\log_4$~~ 4^3 $\qquad \ln e^{-0.5} = -0.5$ $\qquad \log 10^{-4}$

~~$\log_e e$~~ $\cdot -0.5$

3 $\qquad -0.5$ $\qquad -4$

Product Rule of Logarithms
If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b(MN) = \log_b M + \log_b N$$

Write each of the following logarithms as the sum of logarithms.

$\log_2(5 \cdot 3)$

$\log_2 5 + \log_2 3$

$\ln(6z)$

$\ln 6 + \ln z$

Quotient Rule of Logarithms
If M, N and b are positive real numbers, with $b \neq 0$, then

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

Write each of the following logarithms as the difference of logarithms.

$\log_2\left(\frac{5}{3}\right)$

$\log_2 5 - \log_2 3$

$\log\left(\frac{y}{5}\right)$

$\log y - \log 5$

Power Rule of Logarithms
If M and b are positive real numbers, with $b \neq 0$, then

$$\log_b M^r = r \log_b M$$

Use the power Rule of Logarithms to express all powers as factors.

$\log_8 3^5$

$5 \log_8 3$

$3\log_2 5$

$\log_2 5^3$

$\ln x^{\sqrt{3}}$

$\sqrt{3} \ln x$

$\frac{1}{2}\log 16$

$\log 16^{1/2}$

$\log \sqrt{16}$

$\log 4$

Write the following as the sum or difference of logarithms.

$$\log_3\left(\frac{4x}{y}\right)$$

$\log_3 4x - \log_3 y$

$\log_3 4 + \log_3 x - \log_3 y$

$$\log_2(x^2y^3)$$

$\log_2 x^2 + \log_2 y^3$

$2\log_2 x + 3\log_2 y$

Expand the logarithm.

$$\log(8xy^4)$$

$\log 8 + \log x + 4\log y$

$\log 8 + \log x + 4\log y$

$$\log_3\left(\frac{9m^4}{\sqrt[3]{n}}\right)$$

$\log_3 9 + 4\log_3 m - \log_3 \sqrt[3]{n}$

$\log_3 3^2 + 4\log_3 m - \frac{1}{3}\log_3 n$

$2 + 4\log_3 m - \frac{1}{3}\log_3 n$

Write each of the following as a single logarithm.

$\log_6 3 + \log_6 12$

$\log_6(3 \cdot 12)$

$\log_6 36$

$\log_6 6^2$

$\boxed{2}$

$\log(x-2) - \log x$

$\boxed{\log\left(\frac{x-2}{x}\right)}$

Write each of the following as a single logarithm.

$\ln x^5 - 2\ln(xy)$

$\ln x^5 - \ln(xy)^2$

$\ln x^5 - \ln x^2 y^2$

$\ln\left(\frac{x^5}{x^2y^2}\right) = \ln\left(\frac{x^3}{y^2}\right)$

$(x-1)(x+1)$

$\log(x-1) + \log(x+1) - 3\log x$

$\log(x^2-1) - 3\log x$

$\log(x^2-1) - \log x^3$

$\log\left(\frac{x^2-1}{x^3}\right)$

Rewrite and express in terms of a and b (or)
given that a=ln3 and b=ln4

ln36

$\ln(3\cdot3\cdot4)$

$\ln3+\ln3+\ln4$

$a+a+b$

$2\ln4$ $2a+b$

ln27

$\frac{1}{2}\ln144$

Change of Base Formula

If $a \neq 0, b \neq 0,$ and M are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$\log_a M = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

Rewrite the following as a natural log

$\log_4 45 = \frac{\ln 45}{\ln 4}$

$\frac{\log 27}{3} = \frac{1}{3}\log 27$

$\log 27^{1/3}$

$\log 3 = \frac{\ln 3}{\ln 10}$

Use your calculator to approximate the following:

$\log_4 45$ $\log_3 75$ $\log_6 40$

Summary of Properties

$$\log_a a^r = r \qquad b^{\log_b M} = M$$

$$\log_b (MN) = \log_b M + \log_b N$$

$$\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$$

$$\log_b M^r = r \log_b M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$