HW 9-2
Secondary 3H

Name: Key
Date: _____ Period: _____

Evaluate the following expressions by using properties of logarithms

1. $\log_2 2^3 = 3$

2. $3^{\log_3 5} = 5$

3. $e^{\ln 2} = 2$

4. $\log 2 + \log 5 = \log(2 \cdot 5) = \log 10 = 1$

5. $\log_6 2 + \log_6 3$
$\log_6(2 \cdot 3) = \log_6 6 = 1$

Write each expression as a sum and/or difference of logarithms. Write exponents as factors.

6. $\log 1000x^4$
$\log 1000 + \log x^4$
$3 + 4\log x$

7. $\ln(xy^2)$
$\ln x + \ln y^2$
$\ln x + 2\ln y$

8. $\log_4 \sqrt{\frac{x}{y}} = \log\left(\frac{x}{y}\right)^{1/4}$
$= \frac{1}{4}\log\frac{x}{y}$
$= \frac{1}{4}(\log x - \log y) = \frac{1}{4}\log x - \frac{1}{4}\log y$

9. $\log_5(x^2\sqrt{x^2+1})$
$\log_5 x^2 + \log_5 (x^2+1)^{1/2}$
$2\log_5 x + \frac{1}{2}\log_5(x^2+1)$

10. $\log\left(\frac{x^4}{\sqrt[3]{(x-1)}}\right)$
$\log x^4 - \log(x-1)^{1/3}$
$4\log x - \frac{1}{3}\log(x-1)$

Write each expression as a single logarithm

11. $2\ln x + 3\ln y$
$\ln x^2 + \ln y^3$
$\ln x^2 y^3$

12. $\log_4(x+1) - \log_4 x$
$\log_4\left(\frac{x+1}{x}\right)$

13. $\frac{1}{2}\log_3 x + 3\log_3(x-1)$
$\log_3\sqrt{x} + \log_3(x-1)^3$
$\log_3\left(\sqrt{x}(x-1)^3\right)$

14. $\log_8(x^2-1) - \log_8(x+1)$
$\log_8 \frac{x^2-1}{x+1} = \log_8 \frac{(x+1)(x-1)}{x+1}$
$= \log_8(x-1)$

15. $18\log\sqrt{x}+9\log\sqrt[3]{x}-\log 10$

$\log \sqrt{x}^{18} + \log \sqrt[3]{x}^{9} - \log 10$

$\log x^9 + \log x^3 - 1$

$\log x^9 x^3 - 1$

$\log x^{12} - 1$

Use the Change-of-Base to write the following in only natural logarithms. Then use your calculator to evaluate each.

16. $\log_2 10$ $\frac{\log 10}{\log 2} = \frac{\ln 10}{\ln 2} = 3.322$

17. $\log_8 3$ $\frac{\ln 3}{\ln 8} = .528$

18. $\log_{\frac{1}{3}} 19$ $\frac{\ln 19}{\ln \frac{1}{3}} = -2.68$

19. $\log_{\frac{1}{4}} 3$ $\frac{\ln 3}{\ln \frac{1}{4}} = -.792$

20. Show that $\log_a(x+\sqrt{x^2-1})+\log_a(x-\sqrt{x^2-1})=0$

$\log_a (x+\sqrt{x^2-1})(x-\sqrt{x^2-1})$

$\log_a (x^2-(x^2-1))$

$\log_a 1 = 0$ ✓

Write the following in terms of a ~~and~~ or b, given that a=ln2 and b=ln3

21. $\ln 9 = \ln(3\cdot 3) = \ln 3 + \ln 3$

$= 2\ln 3$

$= 2b$

22. $\ln 16 = \ln(2\cdot 2\cdot 2\cdot 2)$

$= \ln 2 + \ln 2 + \ln 2 + \ln 2 = 4\ln 2$

$= 4a$

23. $\frac{1}{3}\ln 27$

$\ln 27^{1/3} = \ln \sqrt[3]{27}$

$= \ln 3$

$= b$

24. $\ln\sqrt{2} = \frac{1}{2}\ln 2$

$= \frac{1}{2}a$

Review

Determine how many terms in the series and then find the sum

1. 3, 6, 12, ..., 12288

$12288 = 3(2)^{n-1}$

$4096 = 2^{n-1}$

$(2)^{12} = 2^{n-1}$

$n = 13$

$\sum_{n=1}^{13} 3(2)^{n-1} =$

24573

2. 5, 15, 45, ..., 98415

$98415 = 5(3)^{n-1}$

$19683 = 3^{n-1}$

$3^9 = 3^{n-1}$

$n = 10$

$\sum_{n=1}^{10} 5(3)^{n-1}$

$= 147{,}620$