

## 9-3 Solving Exponential and Logarithmic equations

## Solving Graphically

$$275e^{0.06x} = 1000$$

$$y_1 = 275e^{0.06x} \quad y_2 = 1000$$

## Solving Equations Algebraically

- simplify any terms possible without using logarithms
- re-write in logarithmic/exponential form
- use the property of equality for logarithmic equations

## Inverses

Addition/Subtraction

Natural Log/e

Common Log/10

$$x - 5 = 10$$

$$e^x = 5$$

$$10^x = 100$$

$$x + 7 = 21$$

$$\ln x = 7$$

$$\log x = 3$$

Solve the following equations

A)  $10 = 5e^{4x}$

$$\frac{10}{5} = \frac{5e^{4x}}{5}$$

$$2 = e^{4x}$$

$$4x = \log_e 2$$

$$\frac{4x}{4} = \frac{\ln 2}{4}$$

C)  $2e^{x-1} + 5 = 80$

$$e^{x-1} = 37.5$$

$$x-1 = \ln(37.5) + 1$$

$$x = 9.62$$

B)  $5^x - 4 = 7$

$$5^x = 11$$

$$x = \log_5 11$$

$$x = 1.49$$

D)  $\frac{20}{20} \left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{5}{20}$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \left(\frac{1}{2}\right)^2$$

$$\frac{x}{3} = 2$$

$$x = 6$$

How long will it take to triple a \$250 initial investment in an account that pays 4.5% compounded quarterly?

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{750}{250} = \frac{250}{250} \left(1 + \frac{.045}{4}\right)^{4t}$$

$$3 = (1.01125)^{4t}$$

$$\frac{4t}{4} = \frac{\log_{1.01125} 3}{1}$$

$$x = 29.6$$

Solve the following

A)  $\ln(x+12) = 3 \ln 2$

$$\ln(x+12) = \ln 8$$

$$x+12 = 8$$

$$x = -4$$

B)  $\log x^4 = 2$

$$10^2 = x^4$$

$$\sqrt[4]{100} = x$$

$$x = 3.16$$

C)  $4 \ln(x+7) - 5 = 1$

$$\frac{4 \ln(x+7)}{4} = \frac{6}{4}$$

$$\ln(x+7) = 1.5$$

$$e^{1.5} = x+7$$

$$e^{1.5} - 7 = -2.52$$

D)  $3 - \log(x+2) = 5$

Solve the following

$$A) \frac{1}{2} \ln(x+3) - \ln x = 0$$

$$\ln \sqrt{x+3} - \ln x = 0$$

$$\ln \left( \frac{\sqrt{x+3}}{x} \right) = 0$$

$$e^0 = \frac{\sqrt{x+3}}{x}$$

$$x \cdot 1 = \frac{\sqrt{x+3} \cdot x}{x}$$

$$x^2 = \sqrt{x+3}^2$$

$$x^2 = x+3$$

$$x^2 - x - 3 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4(1)(-3)}}{2(1)}$$

$$B) \log(x-2) + \log(x+7) = 3 \log 4$$

$$= \frac{1 \pm \sqrt{13}}{2}$$

$$\log(x^2 + 5x - 14) = \log 64$$

$$= \frac{1 + \sqrt{13}}{2}$$

$$x^2 + 5x - 14 = 64$$

$$-64 \quad -64$$

$$x^2 + 5x - 78 = 0$$

## Comparing Earthquake intensities:

On the Richter scale, the magnitude  $M$  of an earthquake depends on the amount of energy,  $E$  (measured in ergs), released by the earthquake as follows:

$$M = \frac{2}{3} \log \frac{E}{10^{11.8}}$$

How much energy is released in a: 7.4 quake compared to a 5.5 quake?

Comparing acidity:  $pH = -\log [H^+]$

$H^+$  hydrogen-ion concentration

Sour Vinegar has a pH of 2.4 and a box of Leg and Sickle baking soda has a pH of 8.4.

a) what are their hydrogen-ion concentrations

b) how many times greater is the  $[H^+]$  of vinegar than baking soda?

c) By how many orders of magnitude do they differ?

## Newton's Law of Cooling

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

This law states that the temperature difference between an object ( $T$ ) and its surroundings ( $T_s$ ) decreases exponentially as a function of time ( $t$ ). Where  $T_0$  is the initial temperature of the object, and  $-k$  is our constant of variation representing the constant rate of decrease in the temperature difference.

A cup of cocoa has cooled from  $95^\circ$  to  $50^\circ$  after 13 minutes in a room at  $25^\circ$ . How long will it take for the cup to cool to  $30^\circ$ ?