9-4 Inverse Functions

Inverse of a Relation

The **inverse of a relation** consisting of the ordered pairs (x, y) is the set of all ordered pairs (y, x).

Notation:

$$f^{-1}(x)$$

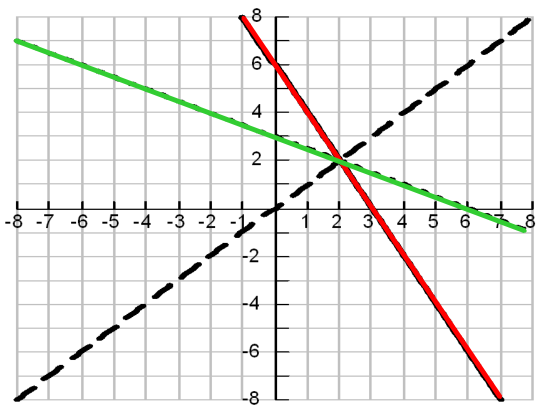
Represents the inverse of the function $f(x)$

Horizontal-Line Test

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function (passed the vertical-line test) at no more than one point.

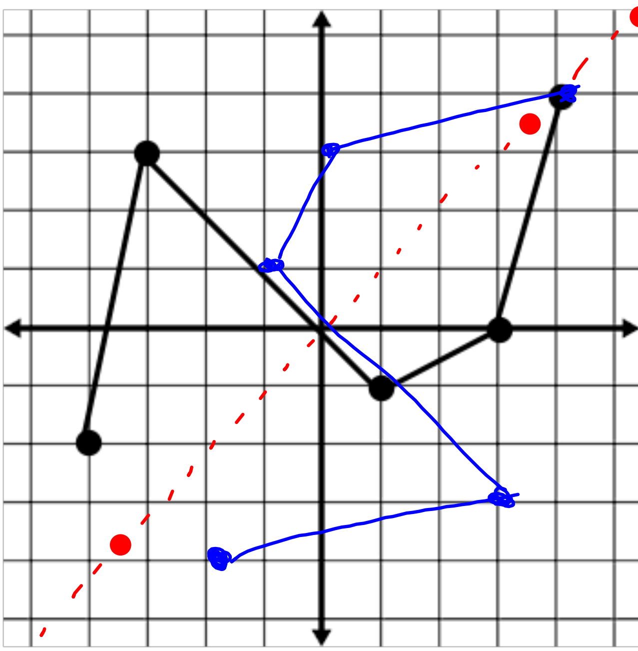
If a function passes both the vertical line test AND the horizontal line test, then it is a **one-to-one** function.

Show $\boxed{f(x) = 6 - 2x}$ and $\boxed{g(x) = \dfrac{6 - x}{2}}$ are inverses graphically.



f(x):	(1,4)	(3,0)	(4,-2)
g(x):	(4,1)	(0,3)	(-2,4)

Graph the inverse of the graph. (Use y=x to find inverse points)



$(-4,-2) \rightarrow$	$(-2,-4)$
$(-3,3) \rightarrow$	$(3,-3)$
$(1,-1) \rightarrow$	$(-1,1)$
$(3,0) \rightarrow$	$(0,3)$
$(4,4) \rightarrow$	$(4,4)$

To find the inverse equation of a function

1. Change $f(x)$ to y.
2. Interchange x and y
3. Solve for new y
4. Change new y to $\boxed{f^{1}(x)}$

Find the inverse of each function. List any domain restrictions if applicable.

$$f(x) = x^2 + 1$$

$$y = x^2 + 1$$

$$x = y^2 + 1$$

$$-1 \qquad -1$$

$$\pm\sqrt{x-1} = \sqrt{y^2}$$

$$y = \pm\sqrt{x-1}$$

$$f^{-1}(x) = \pm\sqrt{x-1}$$

$$g(x) = \frac{x+1}{2x+3}$$

$$2x+3=0$$
$$-3 \quad -3$$
$$\frac{2x}{2} = \frac{-3}{2}$$
$$\boxed{x \neq \frac{-3}{2}}$$

$$y = \frac{x+1}{2x+3}$$

$$(2y+3)\,x = \frac{y+1}{\cancel{2y+3}} \cdot (\cancel{2y+3})$$

$$2xy + 3x = y + 1$$
$$-y \quad -3x \quad -y \quad -3x$$

$$2xy - y = 1 - 3x$$

$$\frac{y(2x-1)}{2x-1} = \frac{1-3x}{2x-1}$$

$$\boxed{y = g^{-1}(x) = \frac{1-3x}{2x-1} \qquad x \neq -\frac{3}{2}, \tfrac{1}{2}}$$

Find the inverse of each function.

$$h(x) = 2x^3 + 3$$

$y = 2x^3 + 3$

$x = 2y^3 + 3$

$\frac{x - 3}{2} = \frac{2y^3}{2}$

$\sqrt[3]{\frac{x-3}{2}} = \sqrt[3]{y^3}$

$\boxed{h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}}$

$$g(x) = \sqrt[3]{x} - 3$$

$y = \sqrt[3]{x} - 3$

$x = \sqrt[3]{y} - 3$

$(x+3)^3 = \sqrt[3]{y}^3$

$(x+3)^3 = y$

$\boxed{g^{-1}(x) = (x+3)^3}$

We can verify that two functions are inverses of each other by determining if the composition of the two functions are both equal to x.

$$f \circ g = \boxed{x} \qquad g \circ f = \boxed{x}$$

$$f \circ f^{-1} = x \qquad f^{-1} \circ f = x$$

Use composition to determine if the following functions are inverses of each other.

$$f(x) = 5x + 1$$

$$g(x) = \frac{x-1}{5}$$

$f \circ g = f(g(x)) = f\left(\frac{x-1}{5}\right) =$

$= \frac{\cancel{5}}{1}\left(\frac{x-1}{\cancel{5}}\right) + 1 = x - 1 + 1$

$= x$ ✓

$g \circ f = g(f(x)) = g(5x+1)$

$= \frac{5x+1-1}{5} = \frac{5x}{5} = x$ ✓