

P6: Complex Numbers

- Obj: Perform operations w/ complex #s.
 ② Use complex conjugates & division

$$a+bi$$

$$\text{Ex: } i^{53} = i^{52} \cdot i \\ = 1(i) = i$$

$$\begin{array}{r} 13 \\ 4 \overline{) 53} \\ \underline{-46} \\ 13 \\ \underline{-12} \\ 1 \end{array}$$

$$i = \sqrt{-1} = i$$

$$i^2 = -1$$

$$i^3 = i^2 \cdot i = -1 \cdot i = -i$$

$$i^4 = i^3 \cdot i^2 = (-i)(-1) = i$$

$$i^5 = i^4 \cdot i = i \cdot i = -1$$

$$i^6 = i^5 \cdot i = (-1) \cdot i = -i$$

$$i^7 = -i$$

$$i^8 = 1$$

$$\text{Ex } (7-3i) + (4+5i)$$

$$\boxed{11+2i}$$

$$\text{Ex } (2-i) - (8+3i)$$

$$\boxed{-6-4i}$$

$$\text{Ex } (2+3i)(5-i)$$

$$10 - 2i + 15i - 3i^2$$

$$10 + 13i - 3i^2$$

$$-3(-1)$$

$$\boxed{13+13i} + 3$$

$$\text{Ex: } z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Find z^2 .

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$\frac{1}{4} + \frac{\sqrt{3}}{4}i + \frac{\sqrt{3}}{4}i + \frac{3}{4}i^2$$

$$-\frac{3}{4}$$

$$-\frac{1}{2} + \frac{2\sqrt{3}}{4}i$$

$$\boxed{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}$$

$$\text{Ex: } (3+2i)^2 = \boxed{5+12i}$$

Complex conjugate:

$$a+bi \xrightarrow{\text{cc}} a-bi$$

Ex. Find the C.C.

$$-3+2i \rightarrow \boxed{-3-2i}$$

$$\overbrace{(-3+2i)(-3-2i)}$$

$$9 \pm \cancel{6i} - \cancel{6i} - \cancel{4i^2} + 4$$

$$\boxed{13}$$

$$\text{Ex: } \frac{(2)}{(1+4i)} \cdot \frac{(1-4i)}{(1-4i)} = \frac{2-8i}{1-\cancel{4i}+\cancel{4i}-\cancel{16i^2}+16}$$

$$\frac{2-8i}{17} = \left(\frac{2}{17} - \frac{8i}{17} \right)$$