Secondary III Quarter 3 Summative Review

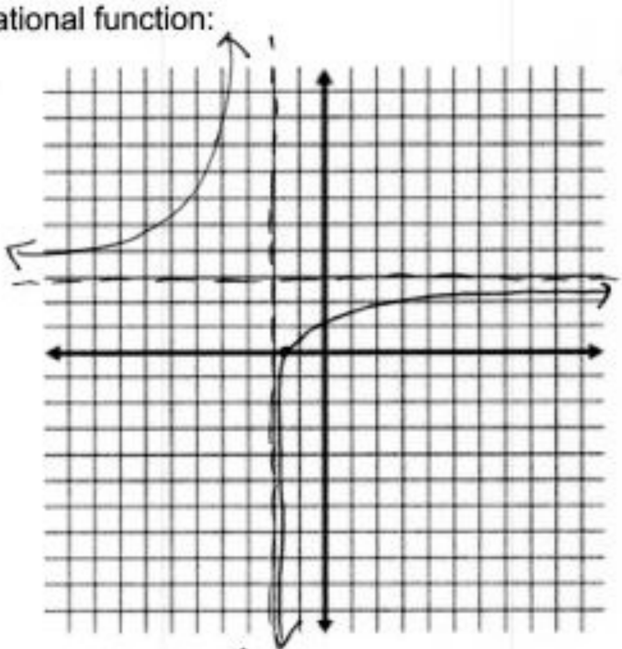
Name: Key

Class: ______

**Unit 7**

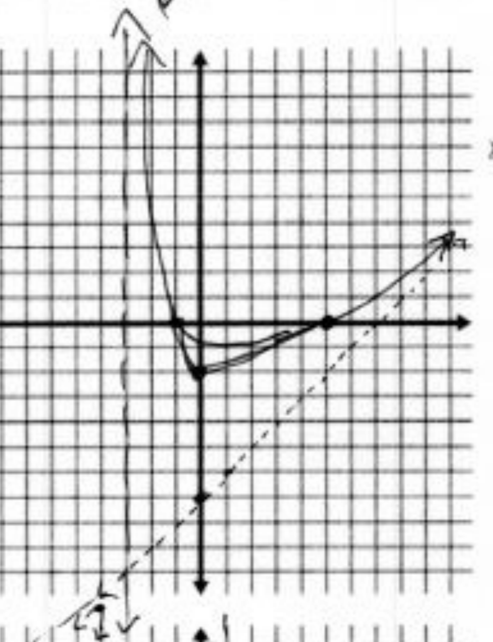
Find the following information and graph each rational function:

1. $f(x)=\dfrac{3x+5}{x+2}$



Domain: $(-\infty,-2)\cup(-2,\infty)$
Range: $(-\infty,3)\cup(3,\infty)$
x- int: $(-5/3,0)$
Vertical Asymptote: $x=-2$
Horizontal Asymptote: $y=3$
Increasing: $(-\infty,-2)\cup(-2,\infty)$
Decreasing: n/a
End Behavior: $\lim_{x\to\pm\infty} f(x)=3$
Asymptote Behavior:
$\lim_{x\to -2^-} f(x)=\infty$ $\lim_{x\to -2^+} f(x)=-\infty$

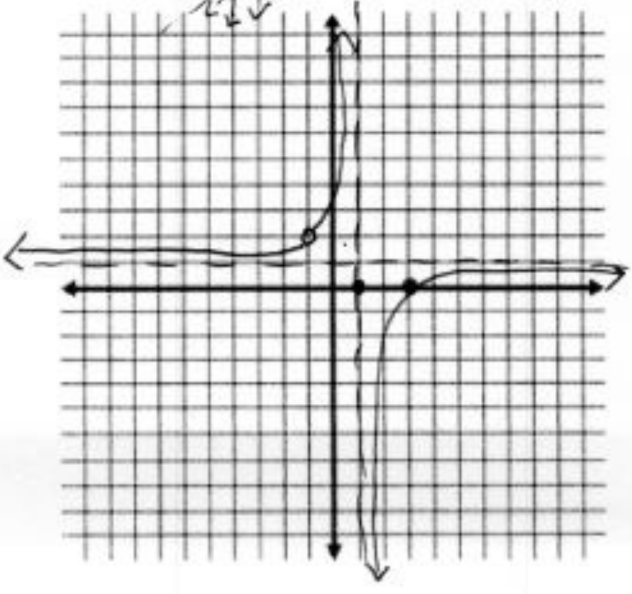
2. $f(x)=\dfrac{x^2-4x-5}{x+3}=\dfrac{(x-5)(x+1)}{(x+3)}$



$$\begin{array}{r} x-7 \\ x+3\overline{)\,x^2-4x-5} \\ -(x^2+3x) \\ \hline -7x-5 \\ -(-7x-21) \\ \hline 16 \end{array}$$

Domain: $(-\infty,-3)\cup(-3,\infty)$
Range: $(-\infty,-18)\cup(-2,\infty)$
x- int: $(5,0)(-1,0)$
Vertical Asymptote: $x=-3$
~~Horizontal~~ Asymptote: no horizontal → $y=x-7$
Increasing: $(-\infty,-7)\cup(1,\infty)$
Decreasing: $(-7,-3)\cup(-3,1)$
End Behavior: $\lim_{x\to\infty} f(x)=\infty$ $\lim_{x\to-\infty} f(x)=-\infty$
Asymptote Behavior:
$\lim_{x\to -3^-} f(x)=-\infty$ $\lim_{x\to -3^+} f(x)=\infty$

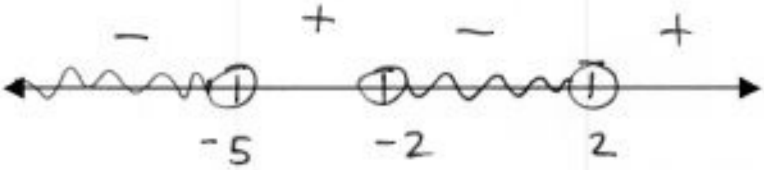
3. $f(x)=\dfrac{x^2-2x-3}{x^2-1}=\dfrac{(x-3)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$



hole: $(-1,2)$
Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$
Range: $(-\infty,1)\cup(1,2)\cup(2,\infty)$
x- int: $(3,0)$
Vertical Asymptote: $x=1$
Horizontal Asymptote: $y=1$
Increasing: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$
Decreasing: n/a
End Behavior: $\lim_{x\to\pm\infty} f(x)=1$
Asymptote Behavior
$\lim_{x\to 1^-} f(x)=\infty$ $\lim_{x\to 1^+} f(x)=-\infty$

Solve the following inequalities using a sign chart:

4. $\frac{x+5}{x^2-4}<0$




$\frac{x+5}{(x+2)(x-2)} < 0$

$-6: \frac{-}{--} = -$

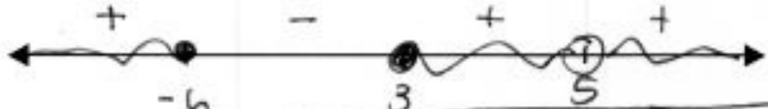
$-3: \frac{+}{--} = +$

$0: \frac{+}{+-} = -$

$3: \frac{+}{++} = +$

$\boxed{(-\infty, -5) \cup (-2, 2)}$

5. $\frac{x^2+3x-18}{x^2-10x+25} \geq 0$




$\frac{(x+6)(x-3)}{(x-5)(x-5)} \geq 0$

$-7: --=+$

$0: +-=-$

$4: ++=+$

$6: ++=+$

$\boxed{(-\infty, -6] \cup [3, 5) \cup (5, \infty)}$

Given the following functions, find all holes, asymptotes, and intercepts.

6. $f(x)=\frac{x-3}{x^2+6x+5}$

$= \frac{x-3}{(x+5)(x+1)}$

holes: none

VA: $x=-5$, $x=-1$

HA: $y=0$

x-int: $(3,0)$

y-int: $(0, -\frac{3}{5})$

7. $f(x)=-\frac{(x^2-4)}{(x+1)} = \frac{-(x+2)(x-2)}{(x+1)}$

holes: n/a

VA: $x=-1$

HA: non horizontal

$y=-x+1$

x-int: $(2,0)(-2,0)$

y-int: $(0,4)$

$$\begin{array}{r|rrr} -1 & -1 & 0 & 4 \\ & \downarrow & 1 & -1 \\ \hline & -1x & +1 & 3 \end{array}$$

8. $f(x)=\frac{x(x+4)^2(x-5)}{(x-5)^2(x+1)^2}$

holes: $x=5$

VA: $x=-1$

~~$x \neq 0$~~

HA: $y=1$

x-int: ~~n/a~~ $(0,0)(-4,0)$

y-int: $(0,0)$

Describe how the graph of *g(x)* is related to the graph $f(x)=\frac{1}{x}$. (transformations)

9. $g(x)=\frac{5}{x}-3$

v. stretch of 5
down 3

10. $g(x)=\frac{-1}{x}+5$

v. reflection
up 5

11. $g(x)=-\frac{1}{(x-2)}+4$

v. reflection
right 2
up 4

12. How do you find the asymptotes (vertical and end behavior) of a rational function?

zeros of denominator

TH: divide numerator by denom.
BH: $y=0$
equal: $\frac{LC}{LC}$

13. Mary and some of her friends are thinking about renting a car while staying at a beach resort for a vacation. The cost per person for staying at the beach resort is \$300, and the cost of the car rental is \$220. If the friends agree to share the cost of the car rental, what is the minimum number of people who must go on the trip so that the total cost for each person is no more than \$350?

$$\frac{220}{x}+300\le 350$$
$$-350 \qquad -350$$

$$\frac{220}{x}-\frac{50x}{x}\le 0 \rightarrow \frac{220-50x}{x}\le 0$$

$220-50x=0$

$\frac{220}{50}=\frac{50x}{50}$

$x=4.4$

$- \quad + \quad -$

$0 \qquad 4.4$

$-1: \frac{+}{-}=-$

$1: \frac{+}{+}=+$

$5: \frac{-}{+}=-$

$[4.4,\infty)$

so at least 5 people must go

14. A basketball team has won 16 games out of 23 games played, for a winning percentage (expressed as a decimal) of $\frac{16}{23}\approx 0.696$. How many consecutive games must the team win to raise its winning percentage to 0.750?

$$\frac{16+x}{23+x}=.750$$

$$16+x=.75(23+x)$$

$$16+x=17.25+.75x$$
$$-16\ -.75x \quad -16 \quad -.75x$$

$$.25x=1.25$$

$x=\boxed{5}$

**Unit 8**

Write an explicit and recursive rule for the following

1. 9, 27, 81, 243,…
1 2 3 4

Explicit: $f(n)=9(3)^{n-1}$

Recursive: $f(1)=9$
$f(n)=3\cdot f(n-1)$

2. 4, −3, −10, −17, ...
1 2 3 4

Explicit: $f(n)=4-7(n-1)$

Recursive: $f(1)=4$
$f(n)=f(n-1)-7$

3. Find the equation that represents exponential decay

a. $y=13(2)^x$ b. $y=\frac{1}{2}(2)^x$ c. $y=2\left(\frac{1}{2}\right)^x$ d. $y=2\left(\frac{4}{2}\right)^x$

4. Find the range of $y=3(2)^{x+3}$ without a calculator.

$(0,\infty)$

Find the stated term for the following sequences

5. −3, −6, −12, −24, ...; 9th term

$f(n)=-3(2)^{n-1}$ $f(9)=-3(2)^{9-1}=\boxed{-768}$

6. What is the y intercept of $y=6\left(\frac{1}{2}\right)^x$?

$(0,6)$

Evaluate the following

7. $\sum_{n=1}^{5} 2n+1$ $=\boxed{35}$

8. $\sum_{k=1}^{3} k^2-1$ $=\boxed{11}$

9. A geometric sequence that has an first term 2, ends with -4374 and has a common ratio of -3, how many terms are in the sequence?

$-4374=2(-3)^{n-1}$
$-2187=(-3)^{n-1}$
$(-3)^7=(-3)^{n-1}$
$7=n-1$
$\boxed{n=8}$

10. For the function $f(x)=5^x$, what is the limit as $x\to\infty$?

A. $\lim_{x\to\infty} f(x)=\infty$ B. $\lim_{x\to\infty} f(x)=-\infty$ C. $\lim_{x\to\infty} f(x)=0$ D. $\lim_{x\to\infty} f(x)=1$

Find the domain, range, and y-intercept for the following functions without graphing. Also state whether the function represents growth or decay.

growth

11. $f(x) = 2(3)^{x-2} - 1$

Domain: $(-\infty, \infty)$
Range: $(-1, \infty)$
y-int: $(0, -7/9)$
Asymptote: $y = -1$

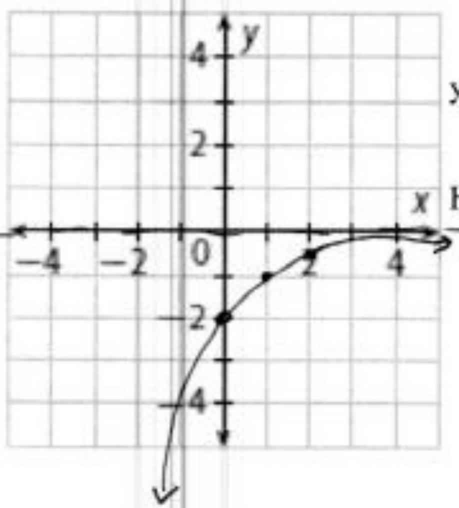
decay

12. $f(x) = \left(\frac{1}{3}\right)^x + 2$

Domain: $(-\infty, \infty)$
Range: $(2, \infty)$
y-int: $(0, 3)$
Asymptote: $y = 2$

Graph the following and label any asymptotes or intercepts

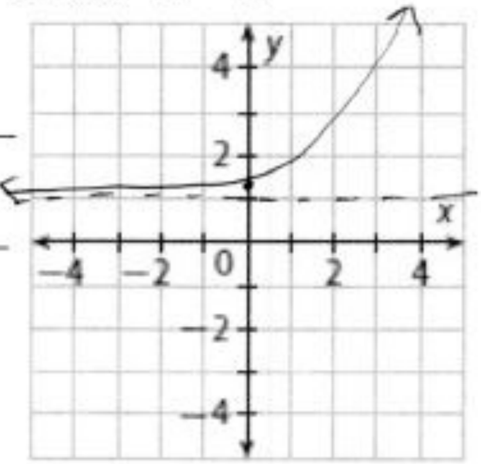
13. $g(x) = -2\left(\frac{1}{2}\right)^x$




y-int $(0, -2)$

HA $y = 0$

14. $f(x) = 2e^{x-2} + 1$




y int $(0, 1.27)$

HA $y = 1$

15. If Jane invests \$4,200 at an 8% interest **compounded continuously**, how much money will there be after 10 years?

$A = 4200e^{.08(10)} = \boxed{\$9{,}347.27}$

(16-18) Answer the following questions with the following: an investment of \$2000 that earns 3.4% interest

16. Write an equation to describe the value V(t) of the investment at time t if the interest is compounded annually.

$$A(t) = 2000\left(1 + \frac{.034}{1}\right)^{t(1)}$$

17. What is the value of the investment after 10 years?

$$\boxed{\$2{,}794.06}$$

18. How long would it take for the investment to reach \$10,000?

$$\frac{10000}{2000} = \frac{2000}{2000}(1 + .034)^t$$

$$5 = 1.034^t$$

$$t = \log_{1.034} 5$$

$$\boxed{t = 48.1 \text{ years}}$$

19. A melting snowman is losing one-half of his weight each day. He originally weighed 128 pounds. Assuming that the outside temperature stays the same, how much does the snowman weigh after 5 days?

$\underset{1}{128}, \underset{2}{64}, \underset{3}{32}, \underset{4}{16}, \underset{5}{8}$

8 pounds

20. A car with a cost of \$25,000 is decreasing in value at a rate of 10% each year. The function $g(t) = 25{,}000(0.9)^t$ gives the value of the car after $t$ years. When will the value of the car be about \$12,000?

$$12000 = 25000(.9)^t$$

$$.48 = .9^t$$

$$t = \log_{.9}(.48)$$

$$\boxed{t = 6.97 \text{ yrs later}}$$

21. An online video game tournament begins with 4096 players. Four players play in each game. In each game there is only one winner, and only the winner advances to the next round. How many games will the winner play?

4096, 1024, 256, 64, 16, 4, 1

winner

$$\boxed{6 \text{ games}}$$

**Unit 9**

Write the given exponential equation as a logarithmic equation

1. $4^2 = 16$ 2. $e^{17} = a$ 3. $10^4 = 10{,}000$ 4. $b^p = a$

$2 = \log_4 16$ $17 = \ln a$ $4 = \log 10000$ $p = \log_b a$

Write the given logarithmic equation as an exponential equation

5. $\log_7 x = 10$ 6. $\ln x = 32$ 7. $\log 1000 = 3$ 8. $\log_\Delta \Phi = \Psi$

$7^{10} = x$ $e^{32} = x$ $10^3 = 1000$ $\Delta^\Psi = \Phi$

9. Evaluate without using a calculator: if $f(x) = \log_5 x$ , find $f(125)$, $f\left(\frac{1}{25}\right)$, $f(\sqrt{5})$

$\log_5 125$ $\log_5 \frac{1}{25}$ $\log_5 \sqrt{5}$

$\boxed{3}$ $\boxed{-2}$ $\boxed{1/2}$

10. Evaluate without using a calculator: if $f(x) = \log_3 x$ find $f(27), f(3), f(\sqrt{3})$

$\boxed{3}$ $\boxed{1}$ $\boxed{1/2}$

Evaluate the following without a calculator:

11. $\log_4 1$ 12. $\ln e$ 13. $\log_5 5$ 14. $7^{\log_7 12}$

0 1 1 12

15. $\log_{12} 12^{15}$ 16. $\ln e^{32}$ 17. $10^{\log 14}$ 17. $\log_5 \sqrt{5}$

15 32 14 $\frac{1}{2}$

Write each as a single logarithm. Assume that all variables are positive.

18. $3\log_4 2+\log_4 6$ $\boxed{\log_4 48}$

19. $\frac{1}{3}\log_7 y-6\log_7 z$ $\boxed{\log \frac{\sqrt[3]{y}}{z^6}}$

20. $(3\log_2 x+\frac{1}{2}\log_2 y)-2\log_2(xz)$

$\log_2 \frac{x^3\sqrt{y}}{x^2 z^2} = \boxed{\log_2 \frac{x\sqrt{y}}{z^2}}$

Use the properties of logarithms to expand the following. Express all exponents as coefficients.

21. $\log_3 x^2y^4$

$2\log_3 x + 4\log_3 y$

22. $\log_{12}\frac{\sqrt{x}}{y^2}$

$\frac{1}{2}\log_{12} x - 2\log_{12} y$

23. $\log_4 \frac{x\sqrt{y}}{z^{12}w^2}$

$\log_4 x + \frac{1}{2}\log_4 y - 12\log_4 z - 2\log_4 w$

Use the Change-of-Base to rewrite the following expressions as natural logarithms.

24. $\log_5 3$

$\frac{\ln 3}{\ln 5}$

25. $\log_{12} 13$

$\frac{\ln 13}{\ln 12}$

26. $\log 80000$

$\frac{\ln 80000}{\ln 10}$

Solve the following. Round your answer to the nearest hundredth. Check for extraneous solutions.

27. $4^{2x+10}+6=262$

$4^{2x+10} = 256$

$2x+10 = \log_4 256$

$2x+10 = 4$

$2x = -6$

$\boxed{x=-3}$

28. $7e^{\frac{x}{4}}=500$

$4 \cdot \frac{x}{4} = \ln\left(\frac{500}{7}\right) \cdot 4$

$\boxed{x = 17.07}$

29. $\log_2 x-\log_2 3=4$

$\log_2 \frac{x}{3} = 4$

$2^4 = x/3$

$\boxed{x=48}$

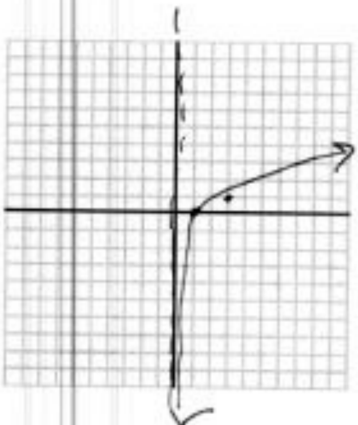
30. $\ln(x-1)=8$

$e^8 = x-1$

$\boxed{x = 2981.96}$

Without a calculator, graph the following, list the transformations (if any), asymptote and two points:

31. $f(x) = \log_3 x$

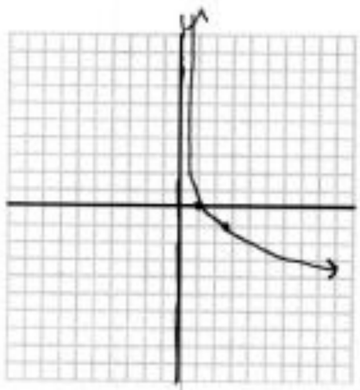


Transformations: none

Points: (1,0) (3,1)

Asymptote: $x = 0$

32. $f(x) = -\ln x$

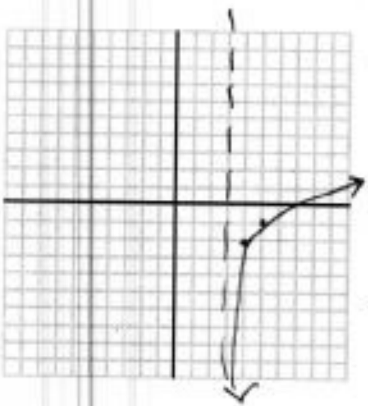


Transformations: v. reflection

Points: (1,0) (2.72, -1)

Asymptote: $x = 0$

33. $f(x) = \log_2(x-3) - 2$

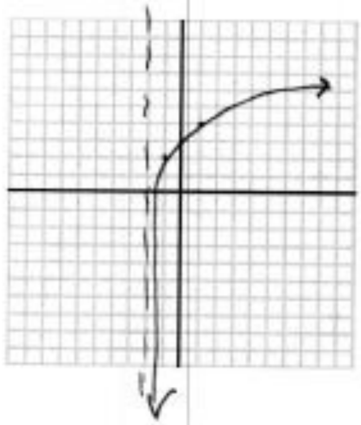


Transformations: right 3 down 2

Points: (4, -2) (5, -1)

Asymptote: $x = 3$

34. $f(x) = 2\log_3(x+2) + 2$



Transformations: v. stretch by 2, left 2, up 2

Points: (-1, 2) (1, 4)

Asymptote: $x = -2$

35. The pH of orange juice is 3.2, and the pH of milk is 6.1.

$pH = -\log[H^+]$ What are the hydrogen-ion concentrations of orange juice and milk?

OJ: $\frac{3.2}{-1} = \frac{-\log[H^+]}{-1}$

$-3.2 = \log[H^+]$

$10^{-3.2} = [H^+]$

$\boxed{6.31 \times 10^{-4}}$

Milk: $6.1 = -\log[H^+]$

$10^{-6.1} = [H^+]$

$\boxed{7.94 \times 10^{-7}}$

36. If Bob invests \$5,000 with a 4% interest rate compounded monthly, how long will it take until his investment has grown to \$7,000? $A = P\left(1+\frac{r}{n}\right)^{nt}$

$\frac{7000}{5000} = \frac{5000}{5000}\left(1 + \frac{.04}{12}\right)^{12t}$

$\frac{7}{5} = \left(1 + \frac{.04}{12}\right)^{12t}$

$\frac{12t}{12} = \frac{\log_{1+\frac{.04}{12}}(7/5)}{12}$

$\boxed{t = 8.426 \text{ years}}$