Verify each identity.

1) $2\cos x \cdot (\cos x - \sin x) = 1 + \cos 2x - \sin 2x$

$2\cos^2 x - 2\sin x \cos x$

$2\left(\dfrac{1 + \cos 2x}{2}\right) - \cancel{2}\sin 2x = 1 + \cos 2x - \cancel{2}\sin 2x$ ✓

2) $\dfrac{\sin^2 x}{1 + \cos 2x} = \dfrac{\tan^2 x}{2}$

$\dfrac{\sin^2 x}{2\cos^2 x} = \dfrac{\tan^2 x}{2}$ ✓

3) $\dfrac{1}{1 - \cos 2x + \sin^2 x} = \dfrac{\csc^2 x}{3}$

$\dfrac{1}{2\sin^2 x + \sin^2 x} = \dfrac{1}{3\sin^2 x} = \dfrac{\csc^2 x}{3}$ ✓

4) $2\sin^2 x + \cot^2 x = \csc^2 x - \cos 2x$

$2\left(\dfrac{1 - \cos 2x}{2}\right) + \csc^2 x - 1$

$x - \cos 2x + \csc^2 x - \cancel{1} = \csc^2 x - \cos 2x$ ✓

5) $\dfrac{\cos x}{\csc^2 x} = \dfrac{\sin^2 x}{\sec x}$

$\cos x \cdot \sin^2 x = \dfrac{\cos \sin^2 x}{\sec x}$ ✓

6) $\dfrac{\tan^2 x}{\cos x} = \dfrac{\sec x}{\cot^2 x}$

$\dfrac{1}{\cot^2 x} \cdot \dfrac{1}{\cos x} = \dfrac{\sec x}{\cot^2 x}$ ✓

7) $\sec^2 x - \csc^2 x = \tan^2 x - \cot^2 x$

$1 + \tan^2 x - (1 + \cot^2 x)$

$1 + \tan^2 x - 1 - \cot^2 x$

$\tan^2 x - \cot^2 x$ ✓

Find the exact value of each.

8) $\sin(195) = \sin(60+135)$

$\sin 60 \, \cos 135 + \cos 60 \, \sin 135$

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{-\sqrt{2}}{2} + \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2}$

$-\dfrac{\sqrt{6}}{4} + \dfrac{\sqrt{2}}{4} = \boxed{\dfrac{\sqrt{2}-\sqrt{6}}{4}}$

9) $\tan(15) = \tan(60-45)$

$= \dfrac{\tan 60 - \tan 45}{1 + \tan 60 \, \tan 45}$

$= \dfrac{\sqrt{3}-1}{1+\sqrt{3}(1)} = \boxed{\dfrac{\sqrt{3}-1}{1+\sqrt{3}}}$

Verify each identity.

10) $\dfrac{\sec^2 x}{\tan^2 x} = \cot^2 x + 1$

$\sec^2 x \cdot \dfrac{1}{\tan^2 x} = \dfrac{1}{\cos^2 x} \cdot \dfrac{\cos^2 x}{\sin^2 x} = \dfrac{1}{\sin^2 x} = \csc^2 x = \cot^2 x + 1$ ✓

11) $\dfrac{\tan x}{1 - \sec^2 x} = -\cot x$

$\dfrac{\tan x}{-\tan^2 x} = \dfrac{1}{-\tan x} = -\cot x$ ✓

12) $\csc x + \tan x \sec x = \dfrac{\csc x}{\cos^2 x}$

$\dfrac{1}{\sin x} + \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\cos x}$

$\dfrac{1}{\sin x} \cdot \dfrac{\cos^2 x}{\cos x} \quad \dfrac{\sin x}{\cos^2 x} \cdot \dfrac{\sin x}{\sin x}$

$\dfrac{\cos^2 x + \sin^2 x}{\cos^2 x \, \sin x} = \dfrac{1}{\cos^2 x \cdot \sin x} = \dfrac{\csc x}{\cos^2 x}$ ✓

13) $\tan(\theta + 135) = \dfrac{\tan\theta - 1}{1 + \tan\theta}$

$\dfrac{\tan\theta + \tan 135}{1 - \tan\theta \tan 135} = \dfrac{\tan\theta + (-1)}{1 - \tan\theta(-1)}$

$= \dfrac{\tan\theta - 1}{1 + \tan\theta}$ ✓

14) $\cos(\theta + 180) = -\cos\theta$

$\cos\theta \cos 180 - \sin\theta \sin 180$

$\cos\theta(-1) - \sin\theta(0)$

$-\cos\theta$ ✓

15) Simplify: $\sec(-x)\cos x$

$\sec x \cdot \cos x$

$\dfrac{1}{\cos x} \cdot \cos x$

$\boxed{1}$

16) Simplify: $\cot(x)\sin(-x)$

$\cot x \cdot -\sin x$

$\dfrac{\cos x}{\sin x} \cdot -\sin x = \boxed{-\cos x}$